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COSMOLOGICAL MODELS BIANCHI TYPE II WITH BULK VISCOSITY IN GENERAL THEORY OF RELATIVITY

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ABSTRACT: - We investigate cosmological models Bianchi type II with bulk viscosity in general theory of relativity. Cosmological models have been obtained by assuming $\eta\theta = \frac{\rho}{3}$ where η is the coefficient of bulk viscosity, θ is the scalar of expansion and ρ is the energy density. In this paper by using relation between metric potential $A = B^m$ further some physical and geometrical properties of the model are discussed. special case for $m = 1$ is also discussed.

KEYWORDS: Expansion scalar, Shear Scalar, Bulk Viscosity, Bianchi Type-II, Space Time.

INTRODUCTION

Cosmology is one of the greatest intellectual achievements of all time beginning from its origin. Cosmology, as a common man under stands is that branch of astronomy, which deals with the large scale structure of the universe.

In the last few years the study of cosmic strings has attracted considerable interest as they are believed to play an important role during early stayes of the universe. The idea was that particles like the photon and the neutron cloud be regarded as waves on a string . The presence of strings in the early universe is a byproduct of grant unified theories (GUT) [1-5] The general relativistic treatment of cosmic strings has been originality given by Letelier [6-7]. Cosmic strings plays an important role in structure formation in cosmology [8] Bianchi type II models play an important role in current modern cosmology for simplification of the actual universe. Krori et al. [7] and Chakraborty and Nandy [9] have investigated Cosmological models for Bianchi type II, VIII and IX space times. Asseo and Sol [10] emphasized the importance of Bianchi type II universe. Patel, Maharaj and Leach [11] have studied the integrability of cosmic string in the context of Bianchi type II, VIII, and IX space times. Bali et al. [12] have investigated string cosmological in general relativity. Rao et al. [13] studied exact Bianchi type II, VIII and IX string cosmological models in Saez – Ballester theory of Gravitation. Recently Wang [14-17] Investigated the LRS Bianchi type III cosmological models for a cloud string with bulk viscosity. Singh and Agarwal [18] studied Bianchi type II, VIII and IX models in scalar tensor theory under the assumption of a relationship between the cosmological constant and scalar field (ψ). Some cosmological solution of massive strings for Bianchi type I space time in presence and absence of magnetic- field have investigated by Banerjee et al. [19]. Roy and Banerjee [20] dealt with LRS Cosmological models of Bianchi type III representing clouds of geometrical as well as massive string Wang [21] has investigated and discussed LRS Bianchi type II space time. A Tyagi and Keerti [22] investigated the Bianchi type II bulk viscous string cosmological models in general relativity. Tiwari and sonia [23] investigated the non-existence of shear in Bianchi type III string cosmological models with bulk viscosity and time dependents Λ term. In 1917 Einstein introduced the cosmological constant into his field equations in order to obtain a static cosmological model since his equations without the cosmological constant admitted only non-static solution.

In this paper, we have investigated cosmological models Bianchi II with bulk viscosity in general theory of relativity. To obtain a determine model we assume that $\eta\theta = \frac{\rho}{3}$ where η is the coefficient of bulk viscosity. We assume that $A = B^m$ where A and B are metric coefficients and m is constant and hence we have discussed solution.

The metric and field equation :

We consider the Bianchi type II space time in the general form.

$$ds^2 = -dt^2 + A^2(dx^2 + dz^2) + B^2\,(dy - xdz)^2 \qquad \text{------------------(1)}$$

Where A and B are functions of time t alone.

The energy momentum tensor for a cloud string with bulk viscous fluid of string is given by

$$T_\alpha^\beta = \rho\, u_\alpha u^\beta - \mu\, x_\alpha x^\beta - \eta u^l_{;l}\,(g_\alpha^\beta + u_\alpha u^\beta) \qquad \text{-------------------------------(2)}$$

Where u_i and x_i satisfy the relations

$$u_\alpha u^\alpha = -x_\alpha x^\alpha = -1\,, u^\alpha\, x_\alpha = 0 \qquad \text{-----------------------------(3)}$$

In equ (2) ρ is the proper energy density for a cloud string with particle attached them, η is the coefficient of bulk viscosity, μ is the string tension density of particles, u^α is the cloud four velocity vector and x^α is a unit space, like vector representing the direction of string. $\theta = u^l_{;l}$ is the scalar of expansion. If the particle density of the configuration is denoted by ρ_p then .

We get.

$$\rho = \rho_p + \mu \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(4)}$$

In co-moving coordinate system we get.

$$u^\alpha = (0,0,0,1),\ x^\alpha = (A^{-1},0,0,0) \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(5)}$$

The Einstein's field equation a system of strings are given by.

$$R_\alpha^\beta - \frac{1}{2}\, R\, g_\alpha^\beta = -T_\alpha^\beta \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(6)}$$

for the metric (1), Einstein's field equation can be written as

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} + \frac{B^2}{4A^4} = \mu + \eta\theta \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(7)}$$

$$\frac{2A_{44}}{A} + \frac{A_4^2}{A} + \frac{3}{4}\,\frac{B^2}{A^4} = \eta\theta \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(8)}$$

$$\frac{A_4^2}{A^2} + \frac{2A_4 B_4}{AB} - \frac{B^2}{4A^4} = \rho \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(9)}$$

Here a suffix '4' indicates an ordinary differentiation with respect to t.

Now there are three independent equation in fine unknown A, B, μ, η and ρ. We need two extra conditions to solve the system completely.

(1a) Relation between η(Coefficient of bulk viscosity), θ (scalar of expansion) and ρ (energy density) [24]

$$\eta\theta = \frac{\rho}{3} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(10)}$$

$$3\eta\theta = \rho \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(11)}$$

(1b) Expansion scalar is proportional to shear scalar $\theta \propto \sigma$ which leads [25,26,27]

$$A = B^m \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(12)}$$

from equ (8), (9), (11) we get

$$\frac{A_{44}}{A} + \frac{A_4^2}{3A^2} - \frac{1}{3}\frac{A_4 B_4}{AB} - \frac{B^2}{3A^4} = 0$$

by using equ (12)

$$B_4 = \frac{1}{\sqrt{4m(m-1)}}^{B^{2-2m}}$$

$$B_4 = \frac{1}{2\sqrt{m(m-1)}}^{B^{2(1-m)}}$$

$$\frac{dB}{dt} = \frac{1}{2\sqrt{m(m-1)}}^{B^{2(1-m)}}$$

Integrating we get

$$B = \frac{1}{2\sqrt{m(m-1)}} \frac{B^{2(1-m)+1}}{2(1-m)+1} + k_1$$

$$B = \frac{1}{2\sqrt{m(m-1)}} \frac{B^{3-2m+1}}{3-2m} + k_1$$

Where k_1 is the constant of integration now

$$\frac{B_4}{B} = \frac{\frac{1}{2\sqrt{m(m-1)}}^{B^{2(1-m)}}}{\frac{1}{2\sqrt{m(m-1)}} \frac{B^{3-2m+1}}{3-2m} + k_1} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_(13)}$$

Taking usual transformation the line element (1) can be written as

$$ds^2 = -\frac{d\tau}{4m(m-1)}\tau^{4(m-1)}\,d\tau + \frac{\tau^{2n}}{4m(m-1)}(dX^2 + dZ^2) + \frac{\tau^2}{4m(m-1)}(dY^2 - XdZ)^2$$

$$ds^2 = -4m(m-1)\tau^{4(m-1)}\,d\tau + \tau^{2n}(dX^2 + dZ^2) + \tau^2(dY - XdZ)^2$$

<u>Some Physical and Geometrical properties</u> :

The energy density ρ, the string tension density μ, Coefficient of bulk viscosity η, the scalar of expansion θ, the shear scalar σ, ρ_p the particle energy density are respectively given by

$$\rho = m(m+2)\left\{\frac{\frac{1}{2\sqrt{m(m-1)}}^{\tau^{2(1-m)}}}{\frac{1}{2\sqrt{m(m-1)}}\frac{\tau^{3-2m}}{3-2m} + k_1}\right\}^2 - \frac{1}{4}\left\{\frac{1}{2\sqrt{m(m-1)}}\frac{\tau^{3-2m}}{3-2m} + k_1\right\}^{2(1-2m)} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_-(15)}$$

$$\mu = \frac{1-m}{2m}\left\{\frac{1}{2\sqrt{m(m-1)}}\frac{\tau^{3-2m}}{3-2m}+k_1\right\}^{2(1-2m)} - 2m(m+1)\left\{\frac{{\frac{1}{2\sqrt{m(m-1)}}}^{\tau^{2(1-m)}}}{\frac{1}{2\sqrt{m(m-1)}}\frac{\tau^{3-2m}}{3-2m}+k_1}\right\}^2 + \left\{\frac{1}{2\sqrt{m(m-1)}}\frac{\tau^{3-2m}}{3-2m}+k_1\right\}^{2(1-2m)} \qquad (16)$$

$$\theta = (2m+1)\frac{\tau_4}{\tau}$$

$$\theta = (2m+1)\,\frac{{\frac{1}{2\sqrt{m(m-1)}}}^{\tau^{2(1-m)}}}{\frac{1}{2\sqrt{m(m-1)}}\frac{\tau^{3-2m}}{3-2m}+k_1} \qquad \mathbf{(17)}$$

$$\sigma = \frac{m-1}{\sqrt{3}}\,\frac{{\frac{1}{2\sqrt{m(m-1)}}}^{\tau^{2(1-m)}}}{\frac{1}{2\sqrt{m(m-1)}}\frac{\tau^{3-2m}}{3-2m}+k_1}$$

$$\eta = \frac{m(m+2)\left\{\frac{{\frac{1}{2\sqrt{m(m-1)}}}^{\tau^{2(1-m)}}}{\frac{1}{2\sqrt{m(m-1)}}\frac{\tau^{3-2m}}{3-2m}+k_1}\right\}^2 - \frac{1}{4}\left\{\frac{1}{2\sqrt{m(m-1)}}\frac{\tau^{3-2m}}{3-2m}+k_1\right\}^{2(1-2m)}}{3(2m+1)\frac{{\frac{1}{2\sqrt{m(m-1)}}}^{\tau^{2(1-m)}}}{\frac{1}{2\sqrt{m(m-1)}}\frac{\tau^{3-2m}}{3-2m}+k_1}} \qquad (18)$$

$$\rho_p = m(m+2)\left\{\frac{{\frac{1}{2\sqrt{m(m-1)}}}^{\tau^{2(1-m)}}}{\frac{1}{2\sqrt{m(m-1)}}\frac{\tau^{3-2m}}{3-2m}+k_1}\right\}^2 - \frac{1}{4}\left\{\frac{1}{2\sqrt{m(m-1)}}\frac{\tau^{3-2m}}{3-2m}+k_1\right\}^{2(1-2m)} - \frac{1-m}{2m}\left\{\frac{1}{2\sqrt{m(m-1)}}\frac{\tau^{3-2m}}{3-2m}+k_1\right\}^{2(1-2m)} - 2m(m+1)\left\{\frac{{\frac{1}{2\sqrt{m(m-1)}}}^{\tau^{2(1-m)}}}{\frac{1}{2\sqrt{m(m-1)}}\frac{\tau^{3-2m}}{3-2m}+k_1}\right\}^2$$

$$+\left\{\frac{1}{2\sqrt{m(m-1)}}\frac{\tau^{3-2m}}{3-2m}+k_1\right\}^{2(1-2m)} \qquad (19)$$

$$\frac{\sigma}{\theta}=\frac{m-1}{\sqrt{3}\;(2m+1)}=\text{k' = constant}$$

CONCLUSION

In this paper we have studied cosmological models Bianchi type II with bulk viscosity. We adopt the condition $\eta\theta=\frac{\rho}{3}$ and $A=B^m$ then the cosmological model with bulk viscosity and cosmological term is obtained. As $\tau\to 0$ the scalar of expansion θ tends to infinitely large and when $\tau\to\infty$ the scalar of expansion $\theta\to 0$. Also at tends to infinity and when $\tau\to\infty$ shear scalar σ tend to zero the energy density $\rho\to\infty$ when $\tau\to 0$ and $\rho\to 0$ when $\tau\to\infty$. Therefore the model describes a shearing, non-rotating, continuously expanding universe with a big bang start. $\frac{\sigma}{\theta}$ = k' = constant therefore model does not approach isotropy for large value of τ. In absence of bulk viscosity i.e. for $\eta=0$ we get $\frac{\sigma}{\theta}=0$ therefore model approach isotropy for large value of τ.

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