ON MATHEMATICAL ANALYSIS FOR BIANCHI TYPE –I STRING COSMOLOGICAL MODEL IN MODIFIED THEORY OF RELATIVITY

R.K. Dubey[1] , Shishir Kumar Srivastava[2], Dhirendra Tripathi [3]
[1]Department of Mathematics Govt. S.K.N.P.G. College, Mauganj (M.P.) India
[2,3]Department of Mathematics APS University Rewa (M.P.) India

ABSTRACT: - We consider the exact solution of Einstein's field equations in a spatially homogenous and anisotropic Bianchi type I string cosmological model in modified theory of relativity with electromagnetic field. To obtain the deterministic solution of Einstein's field equation, by assuming a relation between metric potential B=A^m further some physical and geometrical properties of the model are also discussed.

KEYWORDS: Anisotropic Bianchi type I, string, Electromagnetic field, Cosmology..

INTRODUCTION

Cosmology is the scientific study of large scale properties of the universe as a whole cosmology is study of the motion of crystalline objects. The origin of the universe is greatest cosmological mystery even today. The recent observations that $\wedge \sim 10^{-55}\, cm^{-2}$ while the particle physics predication for $\wedge$ is greater than this value by a factor of order 10^{120}. It is still a challenging problem before us to know the exact physical situation at very early stages of the formation of our universe. The sting theory is a useful concept before the creation of the particle in the universe. The sting are noting but the important topological stable defects due to phase transition that occurs as the temperature lower below some critical temperature at the very early stages of the universe. Bianchi type models are important in the sense that there are homogenous and isotropic from which the process of isotropic is studied through the passage of time these models are tube known as suitable models of our universe, therefore study of Bianchi type models create much more interest.

The string cosmological models with magnetic field in homogenous and anisotropic Bianchi type I, models have been studied by number of researchers in different aspects. Bali [4-5] has obtained Bianchi type I, III, IX string cosmological models in modified theory of relativity. Yadav [1]. Has studied some Bianchi type I viscous fluid string cosmological model with magnetic field. Tripathi and Dubey [2] Have studied LRS Bianchi type I cosmological models with variable deceleration parameter. Bianchi type I universe is also studied with various matters the content of biometric theory of relativity by Deo and Sing [6], Deo and Roughe [7]. The non-titled models have been studied by king and Ellis [3] motivated by aforesaid, we get investigated Bianchi I string cosmological model in modified theory of relativity. We assume a relation B=A^m Where A and B are the metric coefficients and m is constant some physical and geometrical features of the model are also discussed.

The metric and field equation :

We consider the spatially homogenous and anisotropic type I in the form

$$ds^2 = dt^2 - A^2\, dx^2 - B^2 dy^2 - c^2 dz^2 \qquad \text{\_\_\_\_\_\_\_\_\_(1)}$$

Where A,B,C are the metric function of cosmic time 't' only.

$$T_i^j = \rho\, u_i\, u^j - \eta\, x_i\, x^j + E_i^j \qquad \text{\_\_\_\_\_\_\_\_\_(2)}$$

Where, ρ is the rest energy density for a cloud of strings with particles attached along the extension.

We take the equation of state.

$$p = \omega\rho \,, 0 \le \omega \le I \qquad \text{\_\_\_\_\_\_\_\_\_\_(3)}$$

p is it pressure.

$$\rho = \rho_p + \eta \qquad \text{\_\_\_\_\_\_\_\_\_\_(4)}$$

Where ρ_p is particle energy density and η is the tension of the string. u^i and x^i are four velocity vector such that.

$$u_i\, u^i = 1, \quad x_i\, x^i = -1 \text{ and } u_i\, x^i = 0 \qquad \text{\_\_\_\_\_\_\_(5)}$$

u^i is the four velocity vector satisfying the condition.

$$g_{ij}\, u^i\, u^j = 1 \qquad \text{\_\_\_\_\_\_\_(6)}$$

Electromagnetic field is defined as

$$E_i^j = -F_{im}F^{jm} + \frac{1}{4} g_i^j\, F_{\alpha\beta}\, F^{\alpha\beta} \qquad \text{\_\_\_\_\_\_\_\_(7)}$$

Where E_i^j is electromagnetic energy tensor and F_i^j is the electromagnetic field tensor and also we get the Maxwell's equation.

$$F_{ij,k} + F_{jk,i} + F_{ki,j} = 0 \qquad \text{\_\_\_\_\_\_\_\_\_(8)}$$

In the space time equ (8) give us the only non- vanishing components of F_{ij} are F_{12}, F_{13}, F_{24} and F_{34} such that.

$$F_{12} = e\, F_{24}, \quad F_{13} = e\, F_{34} \qquad \text{\_\_\_\_\_\_\_\_\_(9)}$$

Where $e = \pm 1,$ we choose *e*=1 is equ (8) so that it represents an outgoing wave

The Einstein field equation in the general relativity is given by.

$$R_i^j - \frac{1}{2} R g_i^j = -8\pi T_i^j \qquad \text{\_\_\_\_\_\_\_\_(10)}$$

where R_i^j is known as Ricci tensor and $R = g^{ij}R_{ij}$ is the Ricci scalar and T_i^j energy momentum tensor for matter.

In co-moving co-ordinate system for line element (1) and energy momentum tensor (2) to (9), the field equation (10) yields.

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} = 8\,\pi\,k\,\eta \qquad \text{\_\_\_\_\_\_\_\_\_\_\_(11)}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 C_4}{AC} = 8\,\pi\,k \left[\frac{(F_{13})^2}{A^2C^2} - \frac{(F_{12})^2}{A^2B^2}\right] \qquad \text{\_\_\_\_\_\_\_\_\_(12)}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} = -8\,\pi\,k \left[\frac{(F_{13})^2}{A^2C^2} - \frac{(F_{12})^2}{A^2B^2}\right] \qquad \text{\_\_\_\_\_\_\_\_(13)}$$

$$\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{A_4 C_4}{AC} = 8\,\pi\,k\,\rho \qquad \text{\_\_\_\_\_\_\_\_(14)}$$

$$\left[\frac{(F_{13})^2}{A^2C^2} - \frac{(F_{12})^2}{A^2B^2}\right] = 0 \qquad \text{\_\_\_\_\_\_\_\_(15)}$$

Here a suffix '4' indicates an ordinary differentiation with respect to cosmic time t.

The spatral volume for the model is given by.

$$\xi^3_{(t)} = ABC \qquad \text{\_\_\_\_\_\_\_\_(16)}$$

$$\xi = (ABC)^{\frac{1}{3}} \qquad \text{\_\_\_\_\_\_\_\_(17)}$$

Where ξ as the average scale factor, The Hubble parameter and deceleration parameter are respectively defined as

$$H = \frac{\xi_4}{\xi} = \frac{1}{3}\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) \qquad \text{\_\_\_\_\_\_\_\_(18)}$$

$$\mathrm{H} = \frac{1}{3}\left(H_1 + H_2 + H_3\right) \qquad \text{\_\_\_\_\_\_\_\_(19)}$$

Where $H_1 = \frac{A_4}{A}$, $H_2 = \frac{B_4}{B}$, $H_3 = \frac{C_4}{C}$

are the direction Hubble parameter in the x, y, z direction respectinely.

$$g = -\frac{\xi\,\xi_{44}}{\xi_4^2} \qquad \text{\_\_\_\_\_\_\_\_(20)}$$

The Physical quantities of the expansion scalar θ and shear tensor σ^2 are defined as.

$$\theta = 3\,H$$

$$= 3\,\frac{\xi_4}{\xi}$$

$$= 3\,\frac{1}{3}\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)$$

$$\theta = \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) \qquad \text{\_\_\_\_\_\_\_\_(21)}$$

$$\sigma^2 = \sigma_{ij}\sigma^{ij} = \frac{1}{2}\left(\frac{A_4^2}{A^2} + \frac{B_4^2}{B^2} + \frac{C_4^2}{C^2}\right) - \frac{\theta^2}{6} \qquad \text{\_\_\_\_\_\_\_\_(22)}$$

The average anisotropy parameter A_m is given by.

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 \qquad \text{\_\_\_\_\_\_\_\_(23)}$$

Where $\Delta H_i = H_i - \mathrm{H}$ $\qquad (i = 1,2,3)$

represent the directional Hubble parameters in x, y, z direction respectively and $A_m = 0$ corresponds to isotropic expansion.

Solution of field equations :

From equ (15) we get $F_{12} = F_{13} = 0$ i.e. $F_{12} = F_{24} = 0$ & $F_{13} = F_{34} = 0$ ________(24)

Which given us in this theory an isotropic Bianchi type –I cosmological model does not accommodate electromagnetic field.

using equ (24) in (11) to (14) we get a new set of field equ which are.

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{B_4C_4}{BC}=8\,\pi\,k\,\eta \qquad \text{\_\_\_\_\_\_\_\_(25)}$$

$$\frac{A_{44}}{A}+\frac{C_{44}}{C}+\frac{A_4C_4}{AC}=0 \qquad \text{\_\_\_\_\_\_\_\_(26)}$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{AB}=0 \qquad \text{\_\_\_\_\_\_\_(27)}$$

$$\frac{A_4B_4}{AB}+\frac{B_4C_4}{BC}+\frac{A_4C_4}{AC}=8\,\pi\,k\,\rho \qquad \text{\_\_\_\_\_\_\_\_(28)}$$

The system of four field equ (25) to (28) are in five unknown parameters (A,B,C,η,ρ). Therefore, we need extra condition to solve the system completely we assume that expansion scalar is proportional to shear scalar $\theta \propto \sigma$. This condition leads to.

$$B=A^m \qquad \text{\_\_\_\_\_\_\_\_\_\_\_(29)}$$

Where m, is constant.

from equ (27) and (29) we get,

$$(m+1)\,\frac{A_{44}}{A_4}+m^2\,\frac{A_4}{A}=0 \qquad \text{\_\_\_\_\_\_\_\_(30)}$$

integrating we get

$$(m+1)\,log\,A_4+m^2\;logA=log\propto_1$$

$$A_4^{m+1}A^{m^2}=\propto_1 \qquad \text{\_\_\_\_\_\_\_\_(31)}$$

Where $\propto_1$ is constant of integration. Again solving equ (31) we get

$$A=\left(\frac{\propto_2 t+h}{\propto_3}\right)^{\propto_3} \qquad \text{\_\_\_\_\_\_\_\_(32)}$$

Where h is constant of integration.

$$\propto_2^{m+1}=\propto_1,\;\propto_3=\frac{m+1}{m^2+m+1}$$

from equ (29) and (32) we get

$$B=\left(\frac{\propto_2 t+h}{\propto_3}\right)^{m\propto_3} \qquad \text{\_\_\_\_\_\_\_\_(33)}$$

using equ (26) and (27) we get

$$\frac{C_{44}}{C}+\frac{A_4C_4}{AC}=\frac{B_{44}}{B}+\frac{A_4B_4}{AB} \qquad \text{\_\_\_\_\_\_\_\_(34)}$$

Integrating we get

$$(B_4C-CB_4)A=\propto_5 \qquad \text{\_\_\_\_\_\_\_\_(35)}$$

Where $\propto_5$ is constant of integration Again integrating equ (35) we get

$$\frac{C}{B} = \propto_5 \int \frac{dt}{AB^2} + \propto_6 \qquad \text{\_\_\_\_\_\_\_\_(36)}$$

Where $\propto_6$ is a constant of integration.

From equ (32) and (33) in (36) we get

$$C = \frac{\propto_5}{1-(m+1)\propto_3}\left(\frac{\propto_2 t+h}{\propto_3}\right)^{1-(m+1)\propto_3} + \propto_6 \left(\frac{\propto_2 t+h}{\propto_3}\right)^{n\propto_3} \qquad \text{\_\_\_\_\_\_\_\_(37)}$$

Putting the values of A, B, C in equ (1) we get

$$ds^2 = dt^2 - \left(\frac{\propto_2 t+h}{\propto_3}\right)^{2\propto_3} dx^2 - \left(\frac{\propto_2 t+h}{\propto_3}\right)^{2m\propto_3}$$

$$dy^2 - \left[\frac{\propto_5}{1-(m+1)\propto_3}\left(\frac{\propto_2 t+h}{\propto_3}\right)^{1-(m+1)\propto_3} + \propto_6 \left(\frac{\propto_2 t+h}{\propto_3}\right)^{n\propto_3}\right]^2 dz^2 \qquad \text{\_\_\_\_\_\_\_\_(38)}$$

By using a suitable transformation of coordinate, equ(38) becomes

$$ds^2 = \left(\frac{\propto_3}{\propto_2}\right)^2 dT^2 - T^{2\propto_3} dx^2 - T^{2m\propto_3} dy^2 - \left[\frac{\propto_5}{1-(m+1)\propto_3} T^{1-(m+1)\propto_3} + \propto_6 T^{n\propto_3}\right] dz^2 \qquad \text{\_\_\_\_\_\_\_\_(39)}$$

Special model for $\propto_6 = 0$

$$ds^2 = \left(\frac{\propto_3}{\propto_2}\right)^2 dT^2 - T^{2\propto_3} dx^2 - T^{2m\propto_3} dy^2 - \left[\frac{\propto_5}{1-(m+1)\propto_3} T^{1-(m+1)\propto_3}\right]^2 dz^2 \qquad \text{\_\_\_\_\_\_\_(40)}$$

The next energy density ρ, the string tension η and particle density ρ_p for equ (40) given by

$$\rho = 0 \qquad \text{\_\_\_\_\_\_\_(41)}$$

$$8\pi\eta = -\left(\frac{m^2-m-1}{m+1}\right)\frac{\propto_2^2}{T^2} \qquad \text{\_\_\_\_\_\_\_\_(42)}$$

$$8\pi\rho_p = \left(\frac{m^2-m-1}{m+1}\right)\frac{\propto_2^2}{T^2} \qquad \text{\_\_\_\_\_\_\_\_(43)}$$

$$8\pi\eta = -8\pi\rho_p$$

$$8\pi\eta + 8\pi\rho_p = 0$$

$$\eta + \rho_p = 0 \qquad \text{\_\_\_\_\_\_\_\_(44)}$$

That is $\rho = \rho_p + \eta$ is satisfied.

CONCLUSION

In this paper we have presented a new exact solution of solution of Einstein's field equations for Bianchi type I space time string cosmological model. Here We get observed that the spatial volume increases with time t and it becomes infinite for large value of t and ρ, σ, θ all are infinite but vanish for large t. Thus, the model has a big bang singularity at the finite time t and further since $\frac{\sigma}{\theta}$ = constant.

Electromagnetic field does not exists and some physical as well as kinematic properties are discussed and for particular model $\propto_6 = 0$, by using (41) to (43) we see that for cosmic cloud string $\rho = \rho_p + \eta$ is satisfied.

REFERENCES:

1. Yadav. A.K., Pradhan, A. and Singh. A.K. (2012) Bulk viscous Bianchi type I universe with variable G and Astrophysics.
2. Tripathi, S. and Dubey, R.K. (2011) LRS Bianchi type I model with variable deceleration parameter. Indian J. science Res 2(4) ; 79-81
3. King A.R., and Ellis GFR (1973) Tilted homogenous cosmological models. Comm. maths phy [31(3)], 209-242
4. Bali, R., and Sharma, K. (2002) Tilted type I dust fluid cosmological model in general relativity. Pranama J. Phys [58(3)]. 457-463
5. Bali, R, and Anjali (2006) Bianchi type I magnetized string cosmological model in general relativity. Astrophysics model in general relativity. Astrophysics space science Vol. 302.
6. Deo, S.D. and Singh, S.P. (2013) Bianchi type I wet dark universe in biometric relativity. Mathematical theory and modelling vol. 3(9), 30-32.
7. Deo, S.D. and Roughe, A.K (2010) Non Existence of Bianchi type I, V and VI0 cosmic string in Bimetric relativity. Ultra scientist of Physical Sci. Vol. 22 no. 2, 535-538.
8. Bali, R., and Upadhyay, R.D. (2003) LRS Bianchi type I string dust magnetized cosmological models. Astrophys space science vol 283.
9. Bali, R., and Pareek, U.K. (2007) Bianchi type I string dust cosmological model with magnetic field in general relativity.