E- ISSN No: 2395-0269

**International Journal of Applied and Universal Research**
**Volume 2, Issue 1, Jan-Feb. 2015 Available online at: www.ijaur.com**

# RELIABILITY AND AVAILABILITY CHARACTERISTICS OF TWO SYSTEMS HAVING TWO DISSIMILAR COMPONENTS AND TWO SERVICE FACILITIES

**Ruchi Yadav and Dr. Praveen Gupta**
School of studies in statistics Vikram Univesity, Ujjain (M. P.) India

***Abstract:*** In order to improve reliability, two redundant systems are considered. The first system has two dissimilar components working in parallel. The failure time of the components are assumed to be exponentially distributed with different parameters. Failure of one component puts the work pressure on the second component, causing its changed (increased) failure rates. There are two repair facilities to repair the components. The repair time distribution of each server is exponential. Second system differs from the first system due to the additional feature of preventive maintenance. We obtain the expressions for reliability, the mean time to system failure (MTSF) and steady state availability for both the systems.

**Keywords:** Reliability, Availability, Mean time to system failure, Preventive maintenance.

## 1- INTRODUCTION

Two- unit standby system models have been widely studied in the literature of reliability due to their frequent and significant use in modern business and industry. Recently, Mokaddis and Matta (2010), Khaled (2010) and Sharma et.al (2010) have studied two unit standby systems. They have considered a single repair facility to repair both the units. When both the units are failed, one failed unit waits for repair .Researchers in reliability have shown keen interest in the analysis of two (or more) component parallel systems. Owing to the practical utility in modern industrial and technological set-ups of these systems, we come across with the systems in which the failure in one component affects the failure rate of the other component. Taking this concept into consideration, in this paper, two system models are analyzed. Both the systems have two dissimilar components working independently in parallel. In order to prolong the system operation preventive maintenance (inspection, minor repair) is provided in second system at random epochs of time. Several reliability characteristic of interest to system designers and operations managers are obtained.

## 2. SYSTEM DESCRIPTION

I. The system consists of a single unit having two dissimilar components, say A and B arranged in parallel.
II. Failure of one component affects the failure rate of the other component due to increase in working stresses.
III. The system remains operative even if a single component operates.
IV. There are two repair facilities to repair the components. When both the components are failed, they work independently on each component.
V. The repair rates are different, when both the repair facilities work on same component and when both work on different components.
VI. After repair, each component is as good as new.

In Second system the description is same as above and to improve the reliability preventive maintenance is provided at random epochs, when the system is in any operating state.

## 3. NOTATIONS AND STATES OF THE SYSTEM.

E = Set of regenerative States

a = constant failure rate of component A when B is also operating

b = constant failure rate of component B when A is also operating

$\alpha'$ = failure rate of component A when B has already failed

$\beta'$ = failure rate of component B when A has already failed

$\gamma$ = repair rate of component A when B is operating

$\delta$ = repair rate of component B when A is operating

$\theta$ = repair rate of component B when A is also under repair

$\eta$ = repair rate of component A when B is also under repair.

$\mu$ = rate of conducting preventive maintenance

E- ISSN No: 2395-0269
International Journal of Applied and Universal Research
Volume 2, Issue 1, Jan-Feb. 2015 Available online at: www.ijaur.com
JOURNAL OF APPLIED AND UNIVERSAL RESEARCH

$\lambda$ = rate with which system goes for preventive maintenance.

$A_N$ : component A is in normal mode and operative
$B_N$ : component B is in normal mode and operative
$A_R$ : component A is under repair
$B_R$ : component B is under repair
$A_f$ : component A is in failure mode needs repair
$B_f$ : component B is in failure mode needs repair
$A_{NP}$ : component A is under preventive maintenance
$B_{NP}$ : component B is under preventive maintenance.

The system can be in one of the following states:

**First system**

Up states: $S_0$ ($A_NB_N$) , $S_1$ ($A_RB_N$) , $S_2$ ($A_NB_R$)

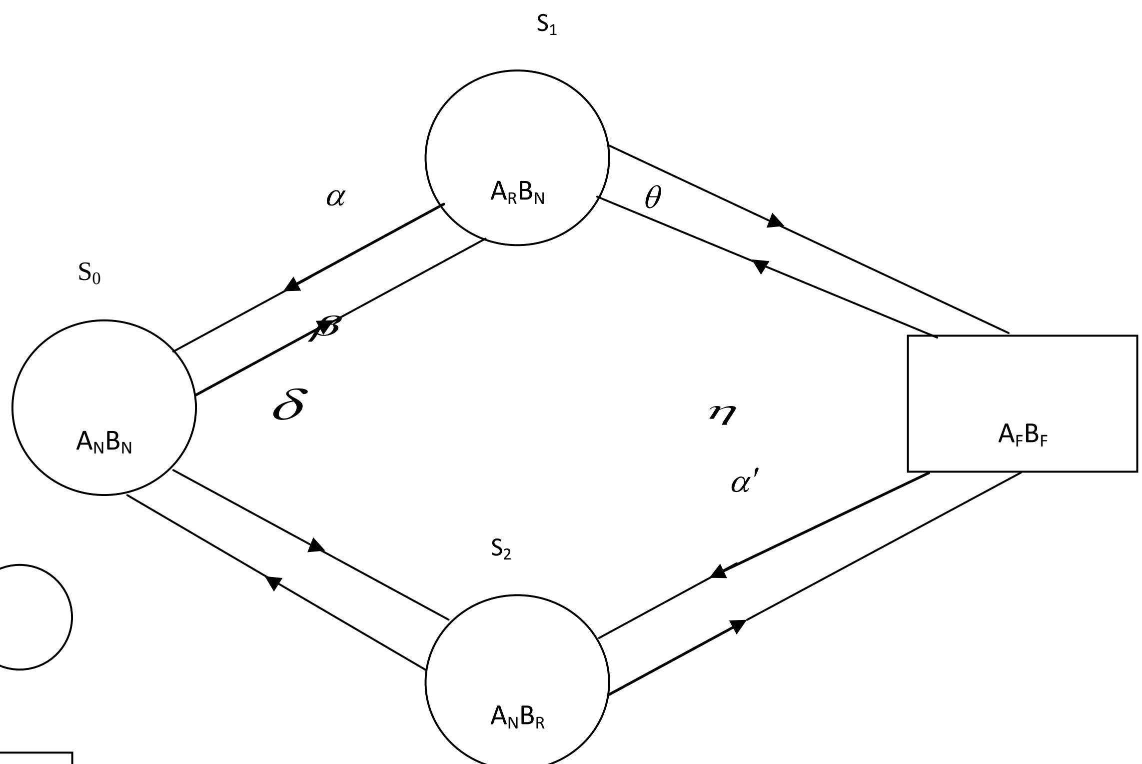
Down states: $S_3$ ($A_FB_F$)

**Second system**

Up states: $S_0$ ($A_NB_N$) , $S_1$ ($A_RB_N$) , $S_2$ ($A_NB_R$)

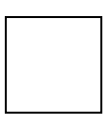
Down states: $S_3$ ($A_FB_F$) , $S_4$ ($A_{NP}B_F$) ,$S_5$ ($A_FB_{NP}$), $S_6$ ($A_{NP}B_{NP}$)

**First System**




Up states:

**Fig1: State Transition Diagram for the first system**

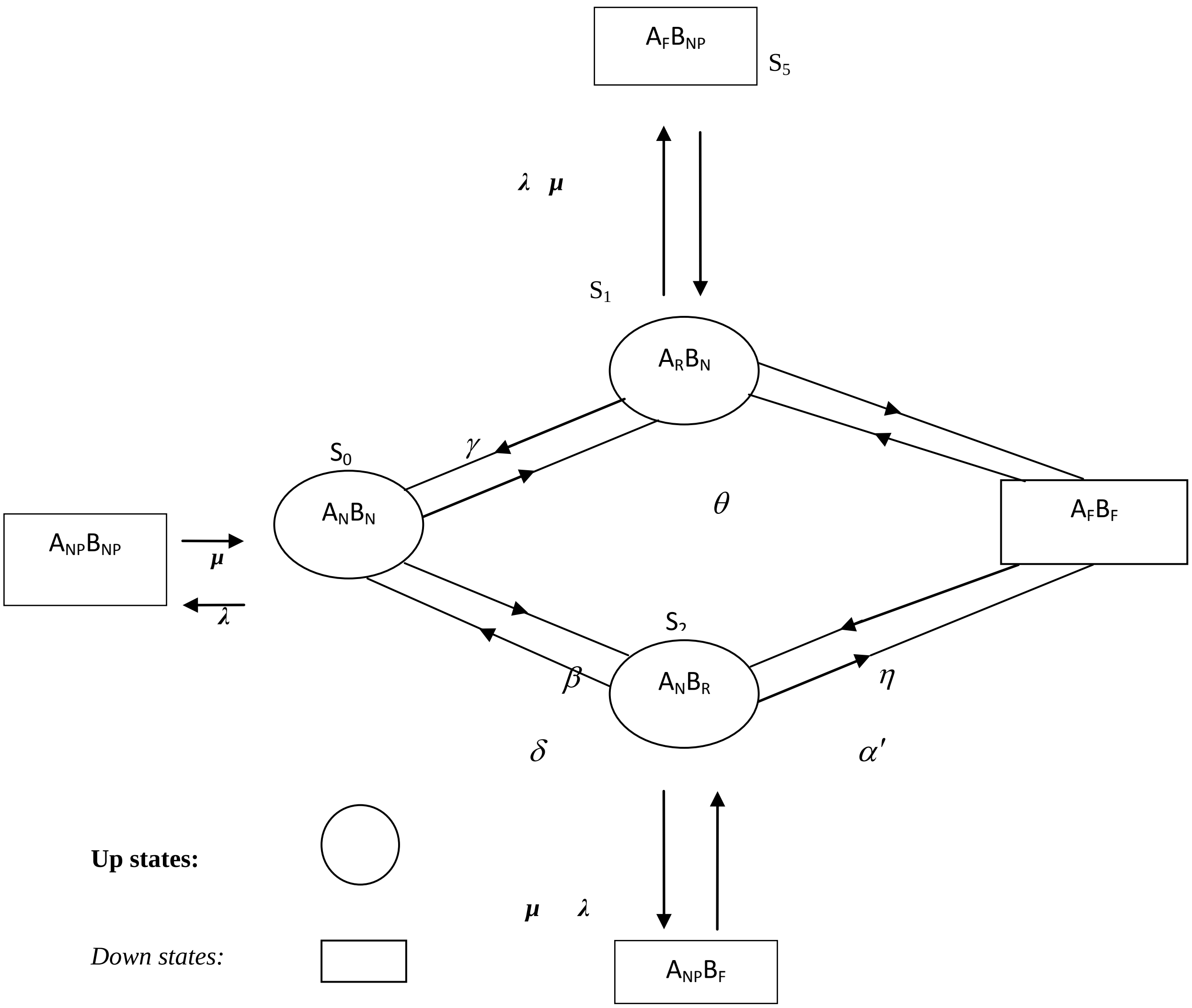
Down state: 

E- ISSN No: 2395-0269

International Journal of Applied and Universal Research
Volume 2, Issue 1, Jan-Feb. 2015 Available online at: www.ijaur.com

Journal OF APPLIED AND UNIVERSAL RESEARCH

**Second System**




**F*ig 2:* State Transition Diagram for the Second System**

E- ISSN No: 2395-0269

International Journal of Applied and Universal Research
Volume 2, Issue 1, Jan-Feb. 2015 Available online at: www.ijaur.com

Journal OF APPLIED AND UNIVERSAL RESEARCH

**1.1.Transition probabilities and sojourn times.**

Let $T_0(=0)$, $T_1$, $T_2$,... be the epochs at which the system enters the state $S_i \in E$, and let $X_n$ denotes the state entered at epoch $T_{n+1}$. i.e. just after the transition of $T_n$.Then $\{X_n, T_n\}$ constitute a Markov- renewal process with the state space E, and

$$Q_{ij}(t) = \Pr[X_{n+1} = S_j, T_{n+1}-T_n \le t \mid x_n = S_i]$$

Then the transition probability matrix of the embedded Markov chain is :

$$P = (P_{ij}) = \lim_{t\to\infty} Q_{ij}(t) = Q(\infty)$$

By simple probabilistic considerations, the non-zero elements of $Q_{ij}(t)$ are: $Q_{01}(t) = \int_0^t \alpha e^{-(\alpha+\beta)u}\,du$

, $Q_{02}(t) = \int_0^t \beta e^{-(\alpha+\beta)u}\,du$ $Q_{10}(t) = \int_0^t \gamma e^{-(\gamma+\beta_1)u}\,du$

, $Q_{13}(t) = \int_0^t \beta_1 e^{-(\gamma+\beta_1)u}\,du$

$$Q_{20}(t) = \int_0^t \delta e^{-(\alpha_1+\delta)u}\,du \quad , \quad Q_{23}(t) = \int_0^t \alpha_1 e^{-(\alpha_1+\delta)u}\,du$$

$$Q_{31}(t) = \int_0^t \theta e^{-(\theta+\eta)u}\,du \quad , \quad Q_{32}(t) = \int_0^t \eta e^{-(\theta+\eta)u}\,du \qquad (1)$$

Taking limit as $t \to \infty$, the steady state transition probabilities $p_{ij}$ can be obtained from (1). Thus

$$P_{ij} = \lim_{t\to\infty} Q_{ij}(t)$$

$$p_{01} = \alpha/(\alpha+\beta) \quad p_{02} = \alpha/(\alpha+\beta) \quad p_{10} = \gamma/(\gamma+\beta_1) \quad p_{13} = \beta_1/(\gamma+\beta_1)$$

$$p_{20} = \delta/(\alpha_1+\delta) \quad p_{23} = \alpha_1/(\alpha_1+\delta) \quad p_{31} = \theta/(\theta+\eta) \quad p_{32} = \eta/(\theta+\eta)$$

From the above probabilities the following relations can be easily verified as:

$$p_{01}+p_{02} = p_{02}+p_{23} = p_{10}+p_{13} = p_{31}+p_{32} = 1.$$

**1.2.MEAN SOJOURN TIMES**

The mean time taken by the system in a particular state $S_i$ before transiting to any other state is known as mean sojourn time and is defined as

$$\mu_i = \int_0^\infty P[T > t]\,dt$$

where T is the time of stay in state $S_i$ by the system. s

To calculate mean sojourn time $m_i$ in state $S_i$ ,we assume that so long as the system is in state $S_i$, it will not transit to any other state. Therefore,

$$\mu_0 = \int_0^\infty e^{-(\alpha+\beta)t}\,dt = 1/(\alpha+\beta), \; \mu_1 = 1/(\gamma+\beta_1), \; \mu_2 = 1/(\alpha_1+\delta), \; \mu_3 = 1/(\theta+\eta). \qquad (2)$$

**1.3.Reliability and Mean Time to System Failure (MTSF).**

To determine $R_i(t)$, the reliability of the system when it starts initially from regenerative state $S_i$ (i= 1,2), We assume the failed state S3 as absorbing. Using simple probabilistic arguments in regenerative point technique, we have

$$R_0(t) = Z_0(t) + q_{01}(t) © R_1(t) + q_{02}(t) © R_2(t)$$

E- ISSN No: 2395-0269
International Journal of Applied and Universal Research
Volume 2, Issue 1, Jan-Feb. 2015 Available online at: www.ijaur.com

$$P_1(t)=Z_1(t)+q_{10}(t)©P_0(t)$$
$$P_2(t)=Z_2(t)+q_{20}(t)©P_0(t), \qquad (3)$$

Where we define $Z_i$ (t) as the probability that starting from state $S_i$ the system remains up till epoch t without passing through any regenerative state.

$$Z_0(t)=e^{-(\alpha+\beta)t}, \quad Z_1(t)=e^{-(\gamma+\beta)t}, \quad Z_2(t)=e^{-(\delta+\alpha)t}$$

Taking Laplace transform of relations and solving, we get

$$P_0^*(s)=\frac{Z_0^*+q_{01}^*Z_1^*+q_{02}^*Z_2^*}{1-q_{01}^*q_{10}^*-q_{02}^*q_{20}^*} \qquad (4)$$

Here for brevity the argument s is omitted .Now by taking the limit as $s \to 0$ in equation (4), the mean time to system failure when the initial state $S_0$, is

$$E(T)=\frac{\mu_0+P_{01}\mu_1+P_{02}\mu_2}{1-P_{01}P_{10}-P_{02}P_{20}} \qquad (5)$$

**1.4.Availability Analysis**.

Let $A_i$ (t) be the probability that starting from state $S_i$ the system is available at epoch t without passing through any regenerative state, Now, obtaining $A_i$ (t) by using elementary probability arguments:

$$A_0(t)=Z_0(t)+q_{01}(t)©A_1(t)+q_{02}(t)©A_2(t)$$
$$A_1(t)=Z_1(t)+q_{10}(t)©A_0(t)+q_{13}(t)©A_3(t)$$
$$A_2(t)=Z_2(t)+q_{20}(t)©A_0(t)+q_{23}(t)©A_3(t)$$
$$A_3(t)=q_{31}(t)©A_1(t)+q_{32}(t)©A_2(t)$$

Taking Laplace transform of above equations and soving for $A_0^*(s)$, by omitting the argument ‘s’ for brevity, we get

$$A_0^*(s)=\frac{N_1(s)}{D_1(s)}$$

Where

$$N_1(s)=\left[Z_0^*+q_{01}^*Z_1^*+q_{02}^*Z_2^*\right]\left[1-q_{13}^*q_{31}^*-q_{32}^*q_{23}^*\right]+\left[q_{01}^*q_{13}^*+q_{02}^*q_{23}^*\right]\left[q_{31}^*Z_1^*+q_{32}^*Z_2^*\right]$$
$$D_1(s)=\left[1-q_{13}^*q_{31}^*-q_{32}^*q_{23}^*\right]\left[1-q_{01}^*q_{10}^*-q_{02}^*q_{20}^*\right]-\left[q_{01}^*q_{13}^*+q_{02}^*q_{23}^*\right]\left[q_{31}^*q_{10}^*+q_{32}^*q_{20}^*\right]$$

Therefore, the steady state availability of the system when its starts operation from $S_0$ is

$$A_0(\infty)=\lim_{t\to\infty}A_0(t)=\lim_{s\to 0} sA_0^*(s)=\frac{N_1(0)}{D_1'(0)}=\frac{N_1}{D_1}$$

Where $N_1$ and $D_1$ are as

$N_1= N_1 (0) = (m_0+P_{01}m_1+P_{02} m_2)(1-P_{13}P_{31}-P_{32}P_{23})+(P_{01}P_{13}+P_{02}P_{23})(P_{31} m_1+P_{32})$ (6)

$D_1 = D_1^1 (0) = (P_{23}P_{32}+P_{01}P_{31})\mu_1 + (P_{32}P_{13}+P_{02}P_{23})\mu_2$ (7)

**E- ISSN No: 2395-0269**

**International Journal of Applied and Universal Research**
**Volume 2, Issue 1, Jan-Feb. 2015 Available online at: www.ijaur.com**

**1.1.Transition probabilities and sojourn Times:**

By simple Probabilistic consideration, the non-zero element of $Q_{ij}(t)$

$$Q_{01}(t)=\alpha\int_0^t e^{-(\alpha+\beta)u}du, \qquad Q_{02}(t)=\beta\int_0^t e^{-(\alpha+\beta)u}du$$

$$Q_{13}(t)=\alpha\int_0^t e^{-(\alpha+\lambda+\gamma)u}du, \qquad Q_{10}(t)=\gamma\int_0^t e^{-(\alpha+\lambda+\gamma)u}du$$

$$Q_{15}(t)=\lambda\int_0^t e^{-(\alpha+\lambda+\gamma)u}du, \qquad Q_{23}(t)=\beta\int_0^t e^{-(\beta+\lambda+\delta)u}du$$

$$Q_{20}(t)=\delta\int_0^t e^{-(\delta+\beta+\lambda)u}du, \qquad Q_{24}(t)=\lambda\int_0^t e^{-(\lambda+\beta+\delta)u}du$$

$$Q_{31}(t)=\eta\int_0^t e^{-(\theta+\eta)u}du, \qquad Q_{42}(t)=Q_{51}=Q_{60}=1 \qquad (1)$$

Taking limit as $t\to\infty$, the steady state transition Probabilities $P_{ij}$'s can be obtained from (1). Thus ,

$$P_{ij}=\lim_{t\to\infty}Q_{ij}(t)$$

$$P_{01}=\alpha/(\alpha+\beta),\quad P_{02}=\beta/(\alpha+\beta),\quad P_{10}=\gamma/(\alpha+\gamma+\lambda),\quad P_{13}=\alpha/(\alpha+\gamma+\lambda)$$

$$P_{15}=\lambda/(\alpha+\beta+\lambda),\quad P_{20}=\delta/(\lambda+\beta+\delta),\quad P_{23}=\beta/(\lambda+\beta+\delta),\quad P_{41}=P_{52}=P_{60}=1 \qquad (2)$$

From the above probabilities the following relations can be easily verified as:

$$P_{01}+P_{02}=1,\quad P_{10}+P_{13}+P_{15}=1,\quad P_{20}+P_{23}+P_{24}=1,\quad P_{41}+P_{52}+P_{60}=1 \qquad (3)$$

**2.2Mean sojourn times**

To calculate mean sojourn time $\mu_i$ in state Si, we assume that so long as the system is in state $S_i$ ,it will not transits to any other state. Therefore,

$$\mu_0=1/(\alpha+\beta),\quad \mu_1=1/(\gamma+\alpha+\lambda),$$
$$\mu_2=1/(\beta+\delta+\lambda),\quad \mu_3=1/(\theta+\eta). \qquad (4)$$

**2.3 Mean time to system failure (MTSF)**

$$R_0(t)=Z_0(t)+q_{01}(t)©R_1(t)+q_{02}(t)©R_2(t)$$
$$R_1(t)=Z_1(t)+q_{10}(t)©R_0(t)$$
$$R_2(t)=Z_2(t)+q_{20}(t)©R_0(t) \qquad (5)$$

where $Z_0(t)=e^{-(\alpha+\beta)t}$, $Z_1(t)=e^{-(\gamma+\alpha+\lambda)t}$, $Z_2(t)=e^{-(\beta+\delta+\lambda)t}$

Taking Laplace transform of relation (5) and solving $R_0^*(s)$ by omitting the argument 's' for brevity, we get

$$R_0^*(s)=\frac{Z_0^*+q_{01}^*Z_1^*+q_{02}^*Z_2^*}{1-q_{01}^*q_{10}^*-q_{02}^*q_{20}^*} \qquad (6)$$

By taking the limit $s\to\infty$ in equation 6

E- ISSN No: 2395-0269
International Journal of Applied and Universal Research
Volume 2, Issue 1, Jan-Feb. 2015 Available online at: www.ijaur.com
Journal OF APPLIED AND UNIVERSAL RESEARCH

Mean time to system failure when the initial state $S_0$,

$$T(\infty)=\frac{\mu_0+P_{01}\mu_1+P_{02}\mu_2}{1-P_{01}P_{10}-P_{02}P_{20}} \qquad (7)$$

**2.4. Availability Analysis**

Now, obtaining $A_i$ (t) by using elementary probability argument;

[illegible]

[illegible]

[illegible]

[illegible]

Taking Laplace transform of above equation and solving $A_0^*(s)$, by omitting the argument 's' for brevity, we get

$$A_0^*(s)=\frac{N_1(s)}{D_1(s)}$$

[illegible]

[illegible]

Therefore, the steady state availability of the system when its starts operation from $S_0$ is

$$A_0(\infty)=\lim_{t\to\infty}A_0(t)=\lim_{s\to 0} sA_0^*(s) = \frac{N_1(0)}{D_1'(0)}$$

Where $N_1$ and $D_1$ are as

$N_1= N_1(0) = (\mu_0+P_{01}\mu_1+P_{02}\mu_2)(1-P_{13}P_{31}-P_{32}P_{23})+(P_{01}P_{13}+P_{02}P_{23})(P_{31}\mu_1+P_{32}\mu_2)$

$D_1 = D_1^1(0) = (P_{20}P_{32}+P_{01}P_{31})\mu_1+(P_{32}P_{13}+P_{02}P_{23})\mu_2$.

**COMPARISION**

**For First system**, the values of MTSF and $A_0$ are obtained for various values of a assuming d= $a^1$ = 2.0, b = 2.5, q = 3.0, $b^1$ = 0.8

| a | 1.0 | 2.0 | 3.0 | 4.0 |
|---|---|---|---|---|
| MTSF | 0.4578 | 0.3286 | 0.2496 | 0.1246 |
| $A_0$ | 0.8156 | 0.7833 | 0.6699 | 0.6322 |

d = $a^1$ = 2.0, a = 2.5, q = 3.0, $b^1$= 0.6

**Considering with variations in b, the values of MTSF and $A_0$ are given below:**

| b | 1.0 | 2.0 | 3.0 | 4.0 |
|---|---|---|---|---|
| MTSF | 0.5586 | 0.4572 | 0.3652 | 0.1246 |
| $A_0$ | 0.9182 | 0.8342 | 0.7699 | 0.6823 |

**For second system , we assume**

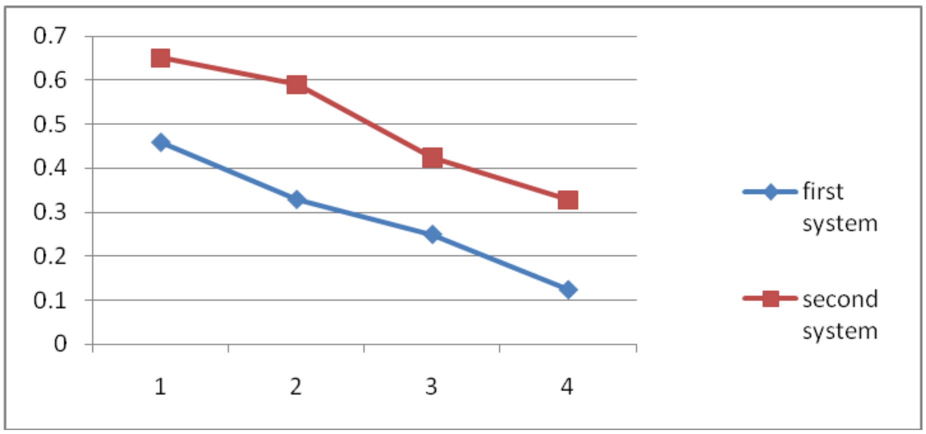
$d = a^1 = 2.0$, $b = 2.5$, $q = 3.0$, $b^1=0.8$, $l = 2.8$, $m = 3$, $\lambda= 2.8$ and vary the values of a

| a | 1.0 | 2.0 | 3.0 | 4.0 |
|---|---|---|---|---|
| MTSF | 0.6576 | 0.5896 | 0.4236 | 0.3276 |
| $A_0$ | 0.9182 | 0.8142 | 0.7696 | 0.6622 |

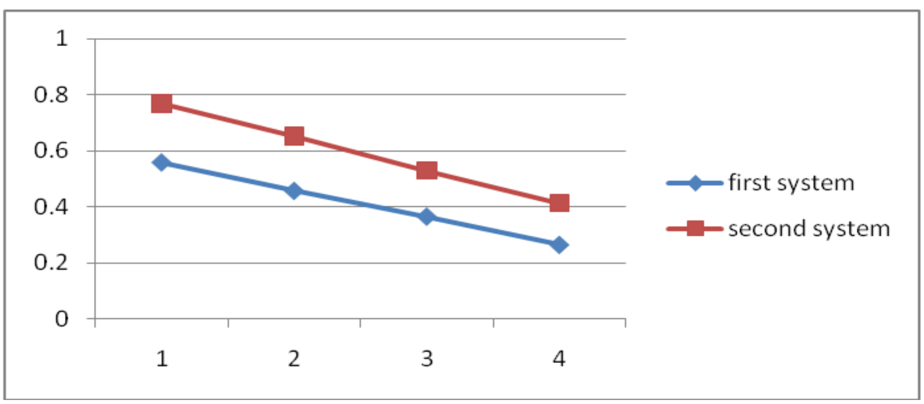
For variation in b and taking

$d = a^1 = 2.0$, $a = 2.5$, $q = 3.0$, $b^1=0.6$, $\lambda= 2.8$, $m = 3$, the values of MTSF and $A_0$ are

| b | 1.0 | 2.0 | 3.0 | 4.0 |
|---|---|---|---|---|
| MTSF | 0.7683 | 0.6528 | 0.5296 | 0.4132 |
| $A_0$ | 0.9957 | 0.8652 | 0.7432 | 0.6153 |



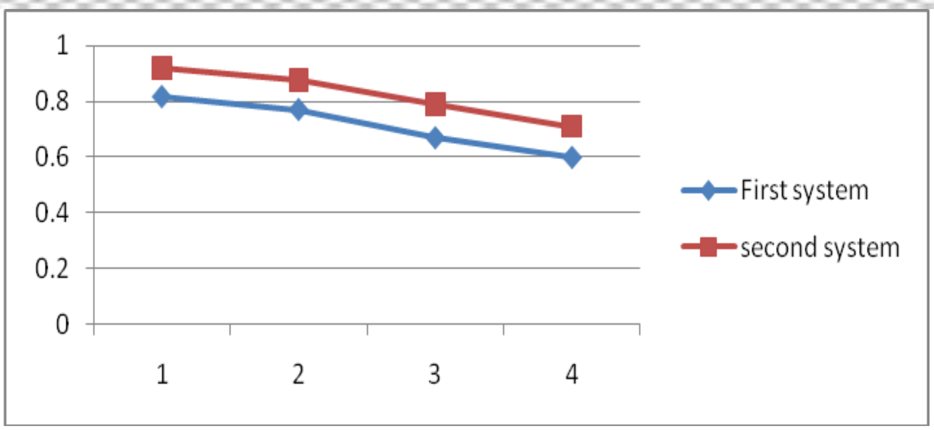

**Comparing MTSF w.r.t. failure rate of component A**



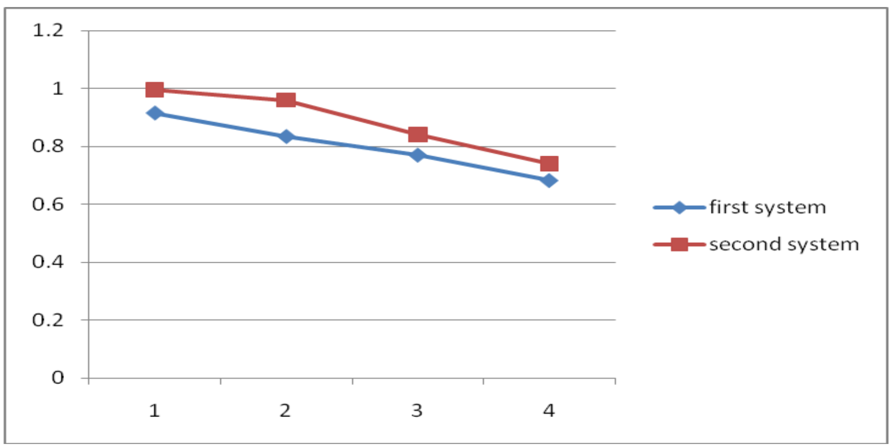

**Comparing MTFS w.r.t. failure rate of component B**

E- ISSN No: 2395-0269

International Journal of Applied and Universal Research
Volume 2, Issue 1, Jan-Feb. 2015 Available online at: www.ijaur.com




**Comparing availability w.r.t failure rate of component A**




**Comparing availability w.r.t. failure rate of component B**

**REFERENCES**

1. Mokaddis, G.S. Matta, C.H. (2010). Cost analysis of a two dissimilar - unit Cold standing redundant system subject to inspection and random change in units, Journal of Mathematics and statistics 6(3), 306-315.
2. Khaleed M. El-said, (2010). Stochastic analysis of a two - unit cold standby system with two stage repair and waiting time. The Indian Journal of Statistics, 72 (B), 188, 1-10.
3. Sharma, S.K. Deepankar Sharma and Vinita Sharma, (2010). Reliability measures for a system having two dissimilar cold standby units with random check and priority repair. Journal of Mathematics and statistics, 2(2) 69-74.
4. Goel, L.R. and Gupta, P. (1984). Stochostic analysis of a two unit parallel system with partial and catastrophic failures and preventive maintenance. Micro electron Reliab, 24 881-883.

E- ISSN No: 2395-0269
International Journal of Applied and Universal Research
Volume 2, Issue 1, Jan-Feb. 2015 Available online at: www.ijaur.com