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BIANCHI TYPE –III ELECTRO MAGNETIZED COSMOLOGICAL MODEL WITH NAMBU STRINGS IN GENERAL THEORY OF RELATIVITY

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ABSTRACT: In this paper we will investigate Bianchi type –III Cosmological model filled with a electro magnetized with Nambu strings in general relativity. Here we assume that in electromagnetic field tensor Eij, E23 is the only non-vanishing component. Under the assumption that the expansion θ in the model is proportional to the shear σ which leads to N = Mn (where M and N are functions of time only). Also we will discuss the cosmological model physical behavior.

KEYWORDS: Bianchi type –III, Nambu strings, Electro Magnetic field.

INTRODUCTION

At times near the beginning stage of the universe symmetry of the universe was broken instinctively during the phase transition. This results into number of stable topological defects; for example, there are issues associated with domain walls and monopoles [1]. These effects are fundamental in the development of cosmological structures as cosmic strings are imperative in the formation of structures during cosmological evolutions [2]. It is well understood that vacuum strings are responsible for density fluctuations which are rather very critical as long as formation of galaxies is concerned [2] as they do have their stress energy in conjunction with the gravitational field.

These gravitational effects later turn out to be of particular research interest. Current arrangement of the universe was never seen in contradiction large scale network of strings during progression of the universe [3], [4], [5]. String theory consists of countless particle type which was later replaced by fundamental building blocks known as strings. Their structure may look like a closed or an open loop but are not limited to any particular shape. Moreover, when these string moves all the way through time they evolve out as a tube or sheet accordingly. Taken into account their important roles string cosmological models has been a topic of great interest for many authors during the last decade.

Relativistic exact solution for Bianchi type string cosmological models were obtained earlier in 1990 [6].

$$T_{ij} = \rho v_i v_j - \lambda x_i x_j + F_{ij} \qquad (1.2)$$

Also class of cosmological solutions of massive strings for Bianchi type VI0 space time has been obtained [7].

In some Bianchi type space times has investigated and discussed some cosmological models and their physical implication [8],[9],[10]. Some cosmologist has presented title electromagnetic Bianchi type III cosmological solution [11]. Presence and absence of magnetic field some obtain exact solutions of massive string of Bianchi Model in space time [12].There are significant observation evidence of Bianchi type –III non-static magnetized cosmological model for perfect fluid circulation in general relativity[13]. Amirhaschi H. Zainuddin [14] suggested Bianchi type III string cosmological models for perfect fluid distribution in general relativity. Pradhan [15] has shown that Massive string cosmology in Bianchi type III space-time with electromagnetic field. Amirhaschi H. Zainuddin [16] studied Magnetized Bianchi type III massive string cosmological model in general relativity. Pradhan and Amirhaschi H. Zainuddin [17] have studied Dark energy model in anisotropic Bianchi type –III space time with variable EoS parameter. Adhav [18] have obtained Bianchi type –III magnetized wet dark fluid cosmological model in general relativity. Upadhaya and Dave [19] take magnetized Bianchi type cosmological model of Massive string with general relativity.

In this paper, we study the progression of anisotropic Bianchi type-III cosmological models in the presence of electromagnetic field. In order to find out the closed form solutions of field equations we consider Nambu strings. In addition, some of the closely associated properties are also resented.

1. THE METRIC AND FIELD EQUATIONS

We take the homogeneous and anisotropic Bianchi type –III metric in the form

$$ds^2 = -dt^2 + L^2\,dx^2 + M^2\,e^{-2\alpha x}dy^2 + N^2 dz^2 \qquad (1.1)$$

Where α is non -zero constant and L, M and N are function of t.

The energy momentum tensor for a cloud of massive string coupled with electromagnetic field of the form

$$T_{ij} = \rho v_i v_j - \lambda x_i x_j + F_{ij} \qquad (1.2)$$

Where, ρ is the rest energy density for a cloud of strings with particles attached along the extension

Thus

$$p = \rho_p + \lambda \qquad (1.3)$$

Where, ρ_ρ is particle energy density and λ is the tension density of the string. v_i - are the four vectors representing the velocity of cloud particles and x_i - the four vectors representing the direction of anisotropy, i.e. z- direction.

Where v_i and x_i satisfy condition

$$v_i v^j = -1, \qquad x_i x^j = 1 \text{ and } v^i x_i = 0 \qquad (1.4)$$

Electromagnetic field is defined as

$$F_{ij} = -E_{ir}E_{jr} + \frac{1}{4}E_{ab}E_{ab}g_{ij} \qquad \textbf{(1.5)}$$

Where, F_{ij} is electromagnetic energy tensor and E_{ij} is the electromagnetic field tensor.

We assume that E_{23} is the only non-vanishing component of E_{ij} which corresponds to the presence of magnetic field along z-direction.

For the line – element (2.1), in a co-moving system, we have

$$T_{11} = \frac{(E_{23})^2 e^{2\alpha x}}{2M^2N^2} \qquad (1.6)$$

$$T_{22} = \frac{-(E_{23})^2 e^{2\alpha x}}{2M^2N^2} \qquad (1.7)$$

$$T_{33} = -\lambda - \frac{(E_{23})^2 e^{2\alpha x}}{2M^2N^2} \qquad (1.8)$$

$$T_{44} = -\rho + \frac{(E_{23})^2 e^{2\alpha x}}{2M^2 N^2} \tag{1.9}$$

All others

$$T_{ij} = 0 \tag{1.10}$$

The Einstein field equation in the general relativity is given by

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi k \, T_{ij} \tag{1.11}$$

Where, R_{ij} is known as Ricci tensor and $R = g_{ij} R_{ij}$ is the Ricci scalar and T_{ij} is energy momentum tensor for matter.

The field equations (1.11) together with the line element (1.1) with equations (1.6) to (1.10) we get

$$\frac{\ddot{M}}{M} + \frac{\ddot{N}}{N} + \frac{\dot{M}\dot{N}}{MN} = -8\pi k\left[\frac{(E_{23})^2 e^{2\alpha x}}{2M^2 N^2}\right] \tag{1.12}$$

$$\frac{\ddot{L}}{L} + \frac{\ddot{N}}{N} + \frac{\dot{L}\dot{N}}{LN} = 8\pi k\left[\frac{(E_{23})^2 e^{2\alpha x}}{2M^2 N^2}\right] \tag{1.13}$$

$$\frac{\ddot{L}}{L} + \frac{\ddot{M}}{M} + \frac{\dot{L}\dot{M}}{LM} - \frac{\alpha^2}{L^2} = 8\pi k\left[\lambda + \frac{(E_{23})^2 e^{2\alpha x}}{2M^2 N^2}\right] \tag{1.14}$$

$$\frac{\dot{L}\dot{M}}{LM} + \frac{\dot{M}\dot{N}}{MN} + \frac{\dot{L}\dot{N}}{LN} - \frac{\alpha^2}{L^2} = 8\pi k\left[\rho - \frac{(E_{23})^2 e^{2\alpha x}}{2M^2 N^2}\right] \tag{1.15}$$

$$\alpha\left(\frac{\dot{M}}{M} - \frac{\dot{L}}{L}\right) = 0 \tag{1.16}$$

Now we have two cases arise

i. $\alpha = 0$
ii. $\alpha \neq 0$

If $\alpha = 0$ and M=N then the first equation be degenerate into Bianchi Type-I. But we consider Bianchi Type –III, so we take up case (ii).

From equation (2.16) become

$$\frac{\dot{M}}{M} - \frac{\dot{L}}{L} = 0 \tag{1.17}$$

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Integrating, we get

$$M = \mu L \tag{1.18}$$

Where μ is a constant of integration

Now we consider Bianchi type –III model with space time, we have L=M, taking μ =1 without loss of generality. Then the field equations (1.12) to (1.15) reduce to

$$\frac{\ddot{M}}{M} + \frac{\ddot{N}}{N} + \frac{\dot{M}\dot{N}}{MN} = -8\pi k\left[\frac{(E_{23})^2 e^{2\alpha x}}{2M^2 N^2}\right] \tag{1.19}$$

$$\frac{\ddot{M}}{M} + \frac{\ddot{N}}{N} + \frac{\dot{M}\dot{N}}{MN} = 8\pi k\left[\frac{(E_{23})^2 e^{2\alpha x}}{2M^2 N^2}\right] \tag{1.20}$$

$$2\frac{\ddot{M}}{M} + \left(\frac{\dot{M}}{M}\right)^2 - \frac{\alpha^2}{M^2} = 8\pi k\left[\lambda + \frac{(F_{23})^2 e^{2\alpha x}}{2B^2 C^2}\right] \tag{1.21}$$

$$\left(\frac{\dot{M}}{M}\right)^2 + 2\frac{\dot{M}\dot{N}}{MN} - \frac{\alpha^2}{M^2} = 8\pi k\left[\rho - \frac{(F_{23})^2 e^{2\alpha x}}{2B^2 C^2}\right] \tag{1.22}$$

From the equation (1.19) and (1.20), we have

$$E_{23} = 0 \tag{1.23}$$

The field equations (1.19) to (1.22) reduce to

$$\frac{\ddot{M}}{M} + \frac{\ddot{N}}{N} + \frac{\dot{M}\dot{N}}{MN} = 0 \tag{1.24}$$

$$2\frac{\ddot{M}}{M} + \left(\frac{\dot{M}}{M}\right)^2 - \frac{\alpha^2}{M^2} = 8\pi k\lambda \tag{1.25}$$

$$\left(\frac{\dot{M}}{M}\right)^2 + 2\frac{\dot{M}\dot{N}}{MN} - \frac{\alpha^2}{M^2} = 8\pi k\rho \tag{1.26}$$

2. SOLUTION OF THE FIELD EQUATIONS

We consider the physically condition which is the expansion of proportional to the shear. This condition leads to

$$N = M^n \tag{2.1}$$

Where n is a constant

I. Nambu Strings

In this case

$$\rho = \lambda \tag{2.2}$$

From equation (1.25), (1.26) and (2.2), we have

$$\frac{\ddot{M}}{M} - \frac{\dot{M}\dot{N}}{MN} = 0 \tag{2.3}$$

Using equation (2.1), the above equation reduce to

$$\frac{\ddot{M}}{M} - n\left(\frac{\dot{M}}{M}\right)^2 = 0 \tag{2.4}$$

Which on integration gives ,

$$M = (1-n)^{\frac{1}{1-n}}(k_1 t + k_2)^{\frac{1}{1-n}} \tag{2.5}$$

Where k_1 and k_2 are constant of integration. Hence we obtain

$$L = (1-n)^{\frac{1}{1-n}}(k_1 t + k_2)^{\frac{1}{1-n}} \tag{2.6}$$

$$N = (1-n)^{\frac{n}{1-n}}(k_1 t + k_2)^{\frac{n}{1-n}} \tag{2.7}$$

Thus the given metric reduces to the form

$$ds^2 = -dt^2 + (1-n)^{\frac{2}{1-n}}(k_1 t + k_2)^{\frac{2}{1-n}}dx^2 + (1-n)^{\frac{2}{1-n}}(k_1 t + k_2)^{\frac{2}{1-n}}e^{-2\alpha x}dy^2 + (1-n)^{\frac{2n}{1-n}}(k_2 t + k_2)^{\frac{2n}{1-n}}dz^2 \tag{2.8}$$

If we transform T = k_1+k_2, $x= X$, $y =Y$, $z=Z$ the metric (3.8) becomes,

$$ds^2 = -\frac{dT^2}{{k_1}^2} + (1-n)^{\frac{2}{1-n}}(T)^{\frac{2}{1-n}}dX^2 + (1-n)^{\frac{2}{1-n}}(T)^{\frac{2}{1-n}}e^{-2\alpha x}dY^2 + (1-n)^{\frac{2n}{1-n}}T^{\frac{2n}{1-n}}dZ^2 \tag{2.9}$$

PHYSICAL AND GEOMETRICAL BEHAVIOUR OF THE MODEL

The rest energy density (ρ)and string tension density (λ) for the model (2.9), we have

$$\rho = \lambda = \frac{1}{8\pi}\left[\frac{(2n+1){k_1}^2}{(1-n)^2T^2} - \frac{\alpha^2}{(1-n)^{\frac{2}{1-n}}(T)^{\frac{2}{1-n}}}\right] \tag{2.10}$$

Using (1.3)

$$\rho_{p=0} \tag{2.11}$$

$$\theta = \frac{k_1(n+2)}{(1-n)T} \tag{2.12}$$

$$\sigma^2 = \frac{(n+2)^2 {k_1}^2}{3(1-n)^2 T^2} \tag{2.13}$$

$$\frac{\sigma^2}{\theta^2} = \frac{1}{3} = constant \tag{2.14}$$

The particle density ρ_p and tension density λ of the cloud string vanish asymptotically in general if (n+2)>0. The expansion in the model stops when n-2 equal to zero. At T=0 the model starts expanding with a big bang and if (n+2)>0 the expansion in the model decreases as time increases . Since $\lim_{T\to\infty} \frac{\sigma}{\theta} \neq 0$, the model does not approach isotropy for large values of t.

CONCLUSION

In this paper we have presented a Bianchi type III cosmological model in electromagnetic field for special cases i.e. Nambu string. It is found that the model always represent accelerating and decelerating universe under the same conditions. It has been shown that in the case of Nambu string, the models are not free from singularities. In present universe we convincingly to say that cosmological model is essential to explicate acceleration. Therefore, with recent observations our theoretical models are in concurrence. In order to solve field equations we have used more general equations of state for the proper rest energy density and string tension density. Since $\frac{\sigma}{\theta}$ is constant in both cases, the models do not approach isotropy at any time. Further the geometrical and physical behavior of the model is also told about that the model does not approach isotropy for the large value of T.

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