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SOME FIXED POINT THEOREMS IN UNIFORM SPACE

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ABSTRACT: In this paper, we define a property called Matric Space property and using this property, we obtain a unique common fixed point for weakly compatible self-mappings of Uniform Space. In this work, we define the nearly uniform convexity and the D-uniform convexity in metric spaces, and prove their equivalence. We also prove the nonlinear version of some classical results related to nearly uniformly convex metric spaces.

KEYWORDS: Fixed point, convexity structure, uniform convexity, uniform normal structure property.

INTRODUCTION

In 1987 [1], Angelov introduced the notion of Φ-contractions on Hausdorff uniform spaces, which simultaneously generalizes the well-known Banach contractions on metric spaces as well as *γ*-contractions [2] on locally convex spaces, and he proved the existence of their fixed points under various conditions. Later in 1991 [3], he also extended the notion of Φ-contractions to *j*-nonexpansive maps and gave some conditions to guarantee the existence of their fixed points. However, there is a minor flaw in his proof of Theorem 1 [3] where the surjectivity of the map *j* is implicitly used without any prior assumption.

In the mathematical field of topology, a **uniform space** is a set with a **uniform structure**. Uniform spaces are topological spaces with additional structure that is used to define uniform properties such as completeness, uniform continuity and uniform convergence.

The conceptual difference between uniform and topological structures is that in a uniform space, one can formalize certain notions of relative closeness and closeness of points. In other words, ideas like "*x* is closer to *a* than *y* is to *b*" make sense in uniform spaces. By comparison, in a general topological space, given sets *A*,*B* it is meaningful to say that a point *x* is *arbitrarily close* to *A* (i.e., in the closure of A), or perhaps that *A* is a *smaller neighborhood* of *x* than *B*, but notions of closeness of points and relative closeness are not described well by topological structure alone. Uniform spaces generalize metric spaces and topological groups and therefore underlie most of analysis.

Definition:

There are three equivalent definitions for a uniform space. They all consist of a space equipped with a uniform structure.

Entourage definition

A nonempty collection *Φ* of subsets $U \subseteq X \times X$ is a uniform structure if it satisfies the following axioms:

1. If $U \in \Phi$, then $\Delta \subseteq U$, where $\Delta = \{(x,x) : x \in X\}$ is the diagonal on $X \times X$.
2. If $U \in \Phi$ and $U \subseteq V$ for $V \subseteq X \times X$, then $V \in \Phi$.
3. If $U \in \Phi$ and $V \in \Phi$, then $U \cap V \in \Phi$.
4. If $U \in \Phi$, then there is $V \in \Phi$ such that $V \circ V \subseteq U$, where $V \circ V$ denotes the composite of V with itself. (The composite of two subsets V and U of $X \times X$ is defined.)
5. If $U \in \Phi$, then $U^{-1} \in \Phi$, where $U^{-1} = \{(y,x) : (x,y) \in U\}$ is the inverse of U.

Properties (2) and (3) state that *Φ* is a filter. If the last property is omitted we call the space **quasiuniform**. The elements U of Φ are called **entourages** from the French word for *surroundings*.

One usually writes $U[x]=\{y : (x,y)\in U\}$. On a graph, a typical entourage is drawn as a blob surrounding the "$y=x$" diagonal; the $U[x]$'s are then the vertical cross-sections. If $(x,y) \in U$, one says that *x* and *y* are *U-close*. Similarly, if all pairs of points in a subset *A* of *X* are *U*-close (i.e., if $A \times A$ is contained in *U*), *A* is called *U-small*. An entourage *U* is *symmetric* if $(x,y) \in U$ precisely when $(y,x) \in U$. The first axiom states that each point is *U*-close to itself for each entourage *U*. The third axiom guarantees that being "both *U*-close and *V*-close" is also a closeness relation in the uniformity. The fourth axiom states that for each entourage *U* there is an entourage *V* that is "not more than half as large". Finally, the last axiom states that the property "closeness" with respect to a uniform structure is symmetric in *x* and *y*.

A **fundamental system of entourages** of a uniformity Φ is any set **B** of entourages of Φ such that every entourage of Φ contains a set belonging to **B**. Thus, by property 2 above, a fundamental systems of entourages **B** is enough to specify the uniformity Φ unambiguously: Φ is the set of subsets of $X \times X$ that contain a set of **B**. Every uniform space has a fundamental system of entourages consisting of symmetric entourages.

Intuition about uniformities is provided by the example of metric spaces: if (X, d) is a metric space, the sets form a fundamental system of entourages for the standard uniform structure of X.

$$U_a = \{(x,y) \in X \times X : d(x,y) \leq a\} \quad \text{where} \quad a > 0$$

Then x and y are U_a-close precisely when the distance between x and y is at most a.

A uniformity Φ is *finer* than another uniformity Ψ on the same set if $\Phi \supseteq \Psi$; in that case Ψ is said to be *coarser* than Φ.

Pseudometrics definition

Uniform spaces may be defined alternatively and equivalently using systems of pseudometrics, an approach that is particularly useful in functional analysis (with pseudometrics provided by seminorms). More precisely, let f: $X \times X \rightarrow \mathbf{R}$ be a pseudometric on a set X. The inverse images $U_a = f^{-1}([0,a])$ for $a > 0$ can be shown to form a fundamental system of entourages of a uniformity. The uniformity generated by the U_a is the uniformity defined by the single pseudometric f. Certain authors call spaces the topology of which is defined in terms of pseudometrics *gauge spaces*.

For a *family* (f_i) of pseudometrics on X, the uniform structure defined by the family is the *least upper bound* of the uniform structures defined by the individual pseudometrics f_i. A fundamental system of entourages of this uniformity is provided by the set of *finite* intersections of entourages of the uniformities defined by the individual pseudometrics f_i. If the family of pseudometrics is *finite*, it can be seen that the same uniform structure is defined by a *single* pseudometric, namely the *upper envelope* sup f_i of the family.

Less trivially, it can be shown that a uniform structure that admits a countable fundamental system of entourages (and hence in particular a uniformity defined by a countable family of pseudometrics) can be defined by a single pseudometric. A consequence is that *any* uniform structure can be defined as above by a (possibly uncountable) family of pseudometrics (see Bourbaki: General Topology Chapter IX §1 no. 4).

Uniform cover definition

A **uniform space** $(X,\mathbf{\Theta})$ is a set X equipped with a distinguished family of coverings **Θ**, called "uniform covers", drawn from the set of coverings of X, that form a filter when ordered by star refinement. One says that a cover **P** is a *star refinement* of cover **Q**, written **P** <* **Q**, if for every $A \in \mathbf{P}$, there is a $U \in \mathbf{Q}$ such that if $A \cap B \neq \emptyset$, $B \in \mathbf{P}$, then $B \subseteq U$. Axiomatically, this reduces to:

{X} is a uniform cover (i.e. $\{X\} \in \mathbf{\Theta}$).

If **P** <* **Q** and **P** is a uniform cover, then **Q** is also a uniform cover.

If **P** and **Q** are uniform covers, then there is a uniform cover **R** that star-refines both **P** and **Q**.

Given a point x and a uniform cover **P**, one can consider the union of the members of **P** that contain x as a typical neighborhood of x of "size" **P**, and this intuitive measure applies uniformly over the space.

Given a uniform space in the entourage sense, define a cover **P** to be uniform if there is some entourage U such that for each $x \in X$, there is an $A \in \mathbf{P}$ such that $U[x] \subseteq A$. These uniform covers form a uniform space as in the second definition. Conversely, given a uniform space in the uniform cover sense, the supersets of $\bigcup\{A \times A : A \in \mathbf{P}\}$, as **P** ranges over the uniform covers, are the entourages for a uniform space as in the first definition. Moreover, these two transformations are inverses of each other.

Topology Uniform Spaces

Every uniform space X becomes a topological space by defining a subset O of X to be open if and only if for every x in O there exists an entourage V such that $V[x]$ is a subset of O. In this topology, the neighborhood filter of a point x is $\{V[x] : V \in \Phi\}$. This can be proved with a recursive use of the existence of a "half-size" entourage. Compared to a general topological space the existence of the uniform structure makes possible the comparison of sizes of neighborhoods: $V[x]$ and $V[y]$ are considered to be of the "same size".

The topology defined by a uniform structure is said to be **induced by the uniformity**. A uniform structure on a topological space is *compatible* with the topology if the topology defined by the uniform structure coincides with the original topology. In general several different uniform structures can be compatible with a given topology on X.

Uniformizable spaces

A topological space is called **uniformizable** if there is a uniform structure compatible with the topology. Every uniformizable space is a completely regular topological space. Moreover, for a uniformizable space X the following are equivalent:

- X is a Kolmogorov space
- X is a Hausdorff space
- X is a Tychonoff space

for any compatible uniform structure, the intersection of all entourages is the diagonal $\{(x, x) : x \text{ in } X\}$.

Some authors (e.g. Engelking) add this last condition directly in the definition of a uniformizable space. The topology of a uniformizable space is always a symmetric topology; that is, the space is an R_0-space.

Conversely, each completely regular space is uniformizable. A uniformity compatible with the topology of a completely regular space X can be defined as the coarsest uniformity that makes all continuous real-valued functions on X uniformly continuous. A fundamental system of entourages for this uniformity is provided by all finite intersections of sets $(f \times f)^{-1}(V)$, where f is a continuous real-valued function on X and V is an entourage of the uniform space **R**. This uniformity defines a topology, which is clearly coarser than the original topology of X; that it is also finer than the original topology (hence coincides with it) is a simple consequence of complete regularity: for any $x \in X$ and a neighbourhood V of x, there is a continuous real-valued function f with $f(x)=0$ and equal to 1 in the complement of V.

In particular, a compact Hausdorff space is uniformizable. In fact, for a compact Hausdorff space X the set of all neighbourhoods of the diagonal in $X \times X$ form the *unique* uniformity compatible with the topology.

A Hausdorff uniform space[*clarification needed*] is metrizable if its uniformity can be defined by a *countable* family of pseudometrics. Indeed, as discussed above, such a uniformity can be defined by a *single* pseudometric, which is necessarily a metric if the space is Hausdorff. In particular, if the topology of a vector space is Hausdorff and definable by a countable family of seminorms, it is metrizable.

Uniform continuity

Similar to continuous functions between topological spaces, which preserve topological properties, are the uniform continuous functions between uniform spaces, which preserve uniform properties. Uniform spaces with uniform maps form a category. An isomorphism between uniform spaces is called a uniform isomorphism.

A uniformly continuous function is defined as one where inverse images of entourages are again entourages, or equivalently, one where the inverse images of uniform covers are again uniform covers.All uniformly continuous functions are continuous with respect to the induced topologies.

Completeness

Generalizing the notion of complete metric space, one can also define completeness for uniform spaces. Instead of working with Cauchy sequences, one works with Cauchy filters (or Cauchy nets).

A **Cauchy filter** F on a uniform space X is a filter F such that for every entourage U, there exists $A \in F$ with $A \times A \subseteq U$. In other words, a filter is Cauchy if it contains "arbitrarily small" sets. It follows from the definitions that each filter that converges (with respect to the topology defined by the uniform structure) is a Cauchy filter. A Cauchy filter is called *minimal* if it contains no smaller (i.e., coarser) Cauchy filter (other than itself). It can be shown that every Cauchy filter contains a unique *minimal Cauchy filter*. The neighbourhood filter of each point (the filter consisting of all neighbourhoods of the point) is a minimal Cauchy filter.

Conversely, a uniform space is called **complete** if every Cauchy filter converges. Any compact Hausdorff space is a complete uniform space with respect to the unique uniformity compatible with the topology.

Complete uniform space enjoy the following important property: if $f: A \to Y$ is a *uniformly continuous* function from a *dense*subset A of a uniform space X into a *complete* uniform space Y, then f can be extended (uniquely) into a uniformly continuous function on all of X.

A topological space that can be made into a complete uniform space, whose uniformity induces the original topology, is called a completely uniformizable space.

Hausdorff completion of a uniform space

As with metric spaces, every uniform space X has a **Hausdorff completion**: that is, there exists a complete Hausdorff uniform space Y and a uniformly continuous map $i: X \to Y$ with the following property: for any uniformly continuous mapping f of X into a complete Hausdorff uniform space Z, there is a unique uniformly continuous map $g: Y \to Z$ such that $f = gi$.

The Hausdorff completion Y is unique up to isomorphism. As a set, Y can be taken to consist of the *minimal* Cauchy filters on$X$. As the neighbourhood filter $\mathbf{B}(x)$ of each point x in X is a minimal Cauchy filter, the map i can be defined by mapping x to$\mathbf{B}(x)$. The map i thus defined is in general not injective; in fact, the graph of the equivalence relation $i(x) = i(x')$ is the intersection of all entourages of X, and thus i is injective precisely when X is Hausdorff.

The uniform structure on Y is defined as follows: for each *symmetric* entourage V (i.e., such that (x,y) is in V precisely when (y,x) is in V), let $C(V)$ be the set of all pairs (F,G) of minimal Cauchy filters *which have in common at least one V-small set*. The sets $C(V)$ can be shown to form a fundamental system of entourages; Y is equipped with the uniform structure thus defined.

The set $i(X)$ is then a dense subset of Y. If X is Hausdorff, then i is an isomorphism onto $i(X)$, and thus X can be identified with a dense subset of its completion. Moreover, $i(X)$ is always Hausdorff; it is called the **Hausdorff uniform space associated with** X. If R denotes the equivalence relation $i(x) = i(x')$, then the quotient space X/R is homeomorphic to $i(X)$.

Example

Every metric space (M, d) can be considered as a uniform space. Indeed, since a metric is *a fortiori* a pseudometric, the pseudometric definition furnishes M with a uniform structure. A fundamental system of entourages of this uniformity is provided by the sets

$$U_a \triangleq d^{-1}([0,a]) = \{(m,n) \in M \times M : d(m,n) \le a\}.$$

This uniform structure on *M* generates the usual metric space topology on *M*. However, different metric spaces can have the same uniform structure (trivial example is provided by a constant multiple of a metric). This uniform structure produces also equivalent definitions of uniform continuity and completeness for metric spaces.

Using metrics, a simple example of distinct uniform structures with coinciding topologies can be constructed. For instance, let $d_1(x,y) = |x - y|$ be the usual metric on **R** and let $d_2(x,y) = |e^x - e^y|$. Then both metrics induce the usual topology on **R**, yet the uniform structures are distinct, since $\{ (x,y) : | x - y | < 1 \}$ is an entourage in the uniform structure for d_1 but not for d_2. Informally, this example can be seen as taking the usual uniformity and distorting it through the action of a continuous yet non-uniformly continuous function.

Every topological group *G* (in particular, every topological vector space) becomes a uniform space if we define a subset *V* of *G* × *G* to be an entourage if and only if it contains the set $\{ (x, y) : x\cdot y^{-1} \text{ in } U \}$ for some neighborhood *U* of the identity element of *G*. This uniform structure on *G* is called the *right uniformity* on *G*, because for every *a* in *G*, the right multiplication $x \to x\cdot a$ is uniformly continuous with respect to this uniform structure. One may also define a left uniformity on *G*; the two need not coincide, but they both generate the given topology on *G*.

For every topological group *G* and its subgroup *H* the set of left cosets *G*/*H* is a uniform space with respect to the uniformity Φ defined as follows. The sets

$$\tilde{U} = \{(s,t) \in G/H \times G/H : \quad t \in U \cdot s\}$$

where *U* runs over neighborhoods of the identity in *G*, form a fundamental system of entourages for the uniformity Φ. The corresponding induced topology on *G*/*H* is equal to the quotient topology defined by the natural map $G \to G/H$.

Main Results

Theorem 3.1. Let X be a real uniformly smooth Banach space with modulus of smoothness of power type $q>1$. Let K be a nonempty, closed convex and bounded subset of X. Suppose $T: K\to (K)$ is a multi-valued quasi-contractive mapping and has fixed point p. Let

(i) $c \ge 1, -1 \ge ckq$,

where c is the constant appearing in (2.1). Let αn and βn real sequences in [0,1] satisfying the condition:

(ii) $\alpha n= \infty,\infty n=1$

(iii) $\lim n\to\infty \beta n = 0$

(iv) $c\alpha nq-1-ckq\beta n\ q-1 \le 1-ckq,$

(v) $c\beta q -1\le 2-ckq2$.

Then for any $x0\ \epsilon$, the sequence $\{xn\}$ defined by **(B)** converges to a fixed point of T.

Proof

Let p be fixed point of T then by using Lemma 2.2 and (B), we have Fixed Point Iterations for Multi-valued Mapping in Uniformly Smooth Banach Space5

$\|xn+1 - p\|q = \|\ 1-\alpha n\ xn+ \alpha nzn\ '- p\|q \le 1 - \alpha n\ q - 1\ \|xn- p\|q+\ c\alpha n\|z''- p\|q\ - \alpha n\ 1-\alpha nq-1\ \|xn- zn'\|q.$ (3.2)

Again $\|zn'- p\|= d\ p,yn \le \max d\ z,Tyn\ z\ \epsilon\ Tp \le H\ Tp,Tyn$.

Therefore,

$\|zn'- p\|q= Hq\ Tp,yn \le kq max\ \|yn- p\|q,dq\ yn,Tyn\ ,dq\ p,Tyn$ 3.2

If dq(p, Tyn) is maximum, then

Hq(Tp,Tyn) ≤ kqdq(p,Tyn ≤kqmax$dq\ z,Tyn\ z\ \epsilon\ Tp$ ≤kqHq Tp,Tyn ,

so that $0 \le \|zn' - p\|q \le Hq(Tp,Tyn) = 0$. Hence, from (3.2),

we get always $\|zn'- p\|\le Kqmax\ \|yn- p\|q,dq\ yn,Tyn\ , \le kq[\|yn- p\|q+dq\ yn,Tyn\ ]$ (3.3) Using Lemma 2.2, (3.3) and (B), we have

$\|yn - p\|q = \|\ 1 - \beta n\ xn + \beta nzn - p\|q \le (1 - \beta n\ q - 1\ \|xn - p\|q + \beta nc\|zn - p\|q - \beta n\ 1- \beta nq-1c\ \|xn - zn\|q$ (3.4)

$dq\ yn,Tyn \le \|yn - zn'\|q \le \|\ 1 - \beta n\ xn + \beta nzn - zn'\|q \le (1-\beta n\ q - 1\ \|xn - zn'\|q + \beta nc\|zn - zn'\|q - \beta n\ 1- \beta nq - 1c\ \|xn - zn\|q.$ (3.5)

Using (3.3),(3.4) and (3.5), we have

$\|zn, - p\|q \le kq\ (1 - \beta n\ q - 1\ \|xn - p\|q+ kq\beta nc\|zn - p\|q + kq\beta nc\|zn - zn'\|q\ kq\ 1 - \beta n\ q - 1\ \|xn - zn'\|q - 2kq\beta n\ 1 - \beta nq- 1\ c\ \|xn - zn\|q.$ (3.6)

Similar to the inequality (3.3), we get that

$\|zn - p\|q = dq\ p,Txn \le Hq\ Tp,Txn \le kq[\|xn - p\|q + dq(xn,Txn)].$ (3.7)

From (3.6) and (3.7), we get

$\|\ zn' - p\|q \le kq\|xn - p\| + kq\ \beta nc\|zn - zn'\|q + kq\ 1 - \beta n\ q-1\ \|xn - zn'\|q - kq\beta n\ 2 - 2\beta nq - 1c - ckq\ dq\ xn,Txn$. (3.8)

From (3.1) and (3.8), we have

$\|xn+1 - p\|q\ 1 - \alpha n\ (q - 1)\ \|xn - p\|q - \alpha n\ (1- \alpha nq - 1c) + c\alpha n[kq\|xn - p\| + kq\beta nc\|zn - zn'\|q + kq\ 1 - \beta n\ q - 1\ \|xn - zn'\|q - kq\beta n\ 2 - 2\beta nq- 1c - ckq\ dq(xn,Txn)] \le 1 - \alpha n\ q - 1- ckq\ \|xn - p\|q\ c2\alpha n\beta nkq\|zn - zn\|q - ckq\alpha n\ \beta n\ 2 - 2\ \beta nq-1c - ckq\ dq\ xn,Txn - \alpha n\ 1- \alpha nq - 1\ ckq\ 1- \beta n\ q - 1\ \|xn - zn'\|$. (3.9)

As $\beta n \to 0$ as $n \to \infty$, there exists a positive integer $N1$ such that $c\beta nq- 1\le 2-ckq2\ \forall\ n \ge N1$

so that

$2 - 2\ \beta nq- 1c - ckq \ge 0\ \forall\ n \ge N1.$

Fixed Point Iterations for Multi-valued Mapping in Uniformly Smooth Banach Space7

Also we have

$\alpha nq-1c-ckq\beta n\ \ q-1\ \ \le 1-ckq,$ Implies that $1-\alpha nq-1c-ck(1-\beta n\ q-1 \ge 0,$ For all $n \ge N1$.

Consequently, from (3.9), we get that for sufficiently large n
‖xn+1 – p‖q≤ 1−αn q−1−ckq ‖xn– p‖q +αnβnc2kqD, (3.10)
Where D is the diameter of C. Now by using lemma 2.3, the sequence {xn} converges top as n → ∞.

Remark 3.2

For Hilbert spaces q=2 and c=1, so that if we set q=2 and c=1 in Theorem 3.1, then condition (q– 1– ckq) reduces to (1– k2) < 1. Moreover, conditions (iii), (iv), and (v) reduce to exactly the same condition of theorem 9 of Sastry and Babu [17].

Theorem 3.3 Let X be a real uniformly smooth Banach space with modulus of smoothness of power type q > 1. Let k be a closed convex and bounded subset of X. Suppose T: K → P (K) is a quasi-contractive and has fixed point p. Let αn be a real sequence satisfying:

(i) i 0≤αn<1$for\ all\ n$≥0
(ii) lim$n$→∞αn=0,
(iii) αn=∞,∞n=0

Then the sequence xn defined by **(A)**, converges to a fixed point of T.

Proof.

By using Lemma 2.2 and (A), we get
‖xn+1 – p‖q=‖ 1−αn xn+αnun– p‖q ≤ 1−αn q−1 ‖xn–p‖q+αnc‖un– p‖q −αn 1−αnq−1λ ‖xn−un‖q (3.11)
Similar to inequality, we get
‖un– p‖q=dq p,Txn ≤Hq Tp,Txn ≤kq‖xn– p‖q+dq xn,Txn . (3.12) Then From (3.11) and (3.12),
we get
‖xn+1 – p‖q(1−αn q1 ‖xn– p‖qαnckq‖xn– p‖q +$c\alpha nkqdq$ xn,Txn −αn 1−αnq−1c ‖un−xn‖q ≤ 1−∝n q−1−ckq ‖xn– p‖q −∝n 1−c∝nq−1−ckq dq xn,Txn . (3.13)
Condition (ii) implies for some N0 sufficient large
N0 ϵ N, 1−αn q−1−ckq < 1−αn q−1−ckq ≥0 so that from (3.13),
we have ‖xn+1 – p‖q≤ 1−∝n q−1−ckq ‖xn– p‖q. (3.14)
Iteration of inequality (3.14) from j=N0 to N yields, ‖xN+1p‖q≤$\boldsymbol{\Pi J}$=$\boldsymbol{N0}$(1−αj(q−1ckq))‖x0– p‖q→0 as n→∞ by condition (iii). Hence xn→∞ as n→∞.

CONCLUSION

We conclude that, by theorems 3.1 and 3.3, either Mann or Ishikawa iterates can be used to approximate the fixed point for multi-valued quasi-contractive mapping in real uniformly smooth Banach space with modulus of smoothness of power type q > 1. (4.2) Theorems 3.1 and 3.3 extends Theorem [6] of Sastry and Babu [5] from Hilbert space to more general Banach space. (4.3) Theorems 3.1 and 3.3 extends Theorems 1 and 2 of Chidume and Osilike [4] from single valued maps to multi-valued maps. Fixed Point Iterations for Multi-valued Mapping in Uniformly Smooth Banach Space 9.

REFERENCES

1. Angelov VG: Fixed point theorems in uniform spaces and applications. Czechoslov. Math. J. 1987, 37: 19–33. 10.1007/BF01597873
2. Cain GL, Nashed MZ: Fixed points and stability for a sum of two operators in locally convex spaces. Pac. J. Math. 1971, 39(3):581–592. 10.2140/pjm.1971.39.581
3. Angelov VG: J -Nonexpansive mappings in uniform spaces and applications. Bull. Aust. Math. Soc. 1991, 43: 331–339. 10.1017/S0004972700029130
4. C.E. Chidume and M. O. Osilike, Fifixed point iterations for quasi-contractive maps in uniformly smooth Banach spaces, Bull Korean Math.Soc.30 (2)(1993), 201–212.
5. K.P.R. Sastry and G.V.R. Babu, convergence of Ishikawa iterates for multi–valued mapping with fixed point, Czechosolovak Math. J. 55 (2005), 817–826.
6. S. Ishikawa, Fixed points by a new iteration method, Proc. Amer. Math. Soc. 44(1) (1974), 147–150.
7. For the year 2007-08, the plan outlay is Rs. 423.39 crores and for XIth five year plan outlay is Rs. 2,256.95 crores is proposed.

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