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ON MATHEMATICAL ANALYSIS FOR BIANCHI TYPE III STRING COSMOLOGICAL MODEL WITH ELECTROMAGNETIC FIELD IN MODIFIED THEORY OF RELATIVITY

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***ABSTRACT**:* - Here We invertigated Bianchi type III string cosmological model with electromagnetic field. To get a determinstic. model F_{23} is only non-vanishing component of Electromagnetic field tensor $F_{\propto B}$. To obtain the determinstic solution of Einstein's field equation by assuming that $\theta \propto \sigma$ which leads to $C = B^m$, The physical and geometrical behaviour of cosmological model are discussed.

KEYWORDS: Anisotropic Bianchi Type III, Cosmic Strings, Electromagnetic field.

INTRODUCTION

The present universe is both spatially homogeneous and isotropic. The basic problem is cosmology is to find the cosmological models of universe and to compare the resulting models with the present day universe using astronomical data. In most treatments of Cosmology, Cosmic fluid is considered as perfect fluid. However, bulk viscosity is expected to play an important role at certain stages of an expanding uinverse. In the last few years of study of cosmic strings has attracted considerable interest as they are believed to paly an important role during early stages of the universe. The idea was that particles like the photon and the neutron could be regarded as waves on a string. The presence of strings in the early universe is a byproduct of Grant unified theories (GUT).

Bianchi type cosmological models are important in the sense that these models are homogeneous and an isotropic, from which the process of isotropization of the universe is studied through the passage of time. It is still a challenging problem before us to know the exact physical situation at very early stages of formation of our universe. The string theory is useful concept before the creation of the particle in the universe. The string are nothing but the important topological stable defects due to the phase transition that occurs as the temperature lower below some critical temperature at the very early stages of the universe. The present day configurations of the universe are not contradicted by the large scale network of strings in the early universe. More over the galaxy formation can be explained by the density fluctuations the vacuum strings.

A. pradhan [1] has presented massive string cosmology in Bianchi type III space time with electromagnetice field . A Pradhan and Amirhaschi H. Zainuddin[2] have studied Dark energy model in an isotropic Bianchi type III space time with Variable EoS parameter. Amirhaschi H. Zainuddin [3] studied Magnetized Bianchi in general relativity. Recently Rathore et al. [4] investigated Bianchi type III string Cosmological models with bulk viscosity and Electromagnetic field. Bali and Jain [5] have studied Bianchi type III non static magnetized cosmological model for perfect fluid distribution in general relativity. Lorenz [6] has presented Tilte electromagnetic Bianchi type III cosmological solution. Tikekar and Patel [7] obtained some exact solutions of massive string of Bianchi type III space time presence and absence of magnetic field, Adhav [8] have obtained Bianchi type III magnetized wet dark fluid cosmological model in general relativity. Upadhya and Dave [9] have investigated some magnetized Bianchi type III Massive string cosmological models in general relativity.

Motivated a for said, we get investigated Bianchi type III string cosmological model with electromagnetic field in modified theory of relativity. To obtain the deterministic solution of Einstein's field equations by assuming that $\theta \propto \sigma$, $C = B^m$ where C and B are the metric coefficients and m is constant. The physical and geometrical behaviour of the model are also discussed.

Metric and field equation:

We consider the spatially homogenous and an isotropic Bianchi type III metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2nx}dy^2 + C^2dz^2 \qquad (1)$$

Where n is non zero constant and A, B, C are the metric funtion of time 't' only.
The energy momentum tensor for a cloud of massive string coupled with electromagnetic field of the form.

$$T_{\propto}^{\beta} = \rho u_{\propto} u^{\beta} - \mu x_{\propto} x^{\beta} + E_{\propto}^{\beta} \qquad (2)$$

Where ρ, is the rest energy density for a cloud of strings with particles attached along the extension
We take the equation of state.

$$p = \omega\rho \quad , \quad 0 \leq \omega \leq 1 \qquad (3)$$

p is it pressure

$$\rho = \rho_p + \eta \qquad (4)$$

Where ρ_p is particle energy density and η is the tension of the string. $u^{\propto}$ and $x^{\propto}$ are four velocity vector such that.

$$u_{\propto} u^{\propto} = 1, \quad x_{\propto} x^{\propto} = -1 \text{ and } u_{\propto} x^{\propto} = 0 \qquad (5)$$

$u^{\propto}$ is the four velocity vector satisfying the condition.

$$g_{\propto\beta} u^{\propto} u^{\beta} = 1 \qquad (6)$$

Electromagnetic field is defined as.

$$E_{\propto}^{\beta} = -F_{\propto m} F^{\beta m} + \frac{1}{4} g_{\propto}^{\beta} F_{\gamma s} F^{\gamma\delta} \qquad (7)$$

Where, $E_{\propto}^{\beta}$ is electromagnetic energy tensor and $E_{\propto}^{\beta}$ is the electromagnetic field tensor. we assume that F_{23} is the only non-vanishing component of $F_{\propto\beta}$ which corresponds to the presence of magnetic field along Z- direction for the line-element (1), in a co-moving co-ordinate system we get.

$$T_1^1 = \frac{(F_{23})^2 e^{2nx}}{2B^2C^2} \qquad (8)$$

$$T_2^2 = \frac{-(F_{23})^2 e^{2nx}}{2B^2C^2} \qquad (9)$$

$$T_3^3 = -\eta - \frac{(F_{23})^2 e^{2nx}}{2B^2C^2} \qquad (10)$$

$$T_4^4 = -\rho + \frac{(F_{23})^2 e^{2nx}}{2B^2C^2} \qquad (11)$$

The Einstein field equation in the general relativity is given by

$$R_{\propto}^{\beta} - \frac{1}{2} R g_{\propto}^{\beta} = -8\pi k T_{\propto}^{\beta} \qquad (13)$$

Where $R_{\propto}^{\beta}$ is known as Ricci tensor and $R = g^{\propto\beta} R_{\propto\beta}$ is the Ricci scalar and $T_{\propto}^{\beta}$ energy momentum tensor for matter The field equation (13) together with the line element (1) with equation (8) to (12) we get

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} = -8\pi G \left[\frac{(F_{23})^2 e^{2nx}}{2B^2C^2}\right] \qquad (14)$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} = 8\pi G \left[\frac{(F_{23})^2 e^{2nx}}{2B^2C^2}\right] \qquad (15)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{n^2}{A^2} = 8\pi G \left[\eta + \frac{(F_{23})^2 e^{2nx}}{2B^2C^2}\right] \qquad (16)$$

$$\frac{A_4 B_4}{AB} + \frac{B_4 C_4}{BC} + \frac{A_4 C_4}{AC} - \frac{n^2}{A^2} = 8\pi G \left[\rho - \frac{(F_{23})^2 e^{2nx}}{2B^2C^2}\right] \qquad (17)$$

$$n\left(\frac{B_4}{B} - \frac{A_4}{A}\right) = 0 \qquad (18)$$

Here a suffix '4' indicates an ordinary differentiation with resect to time 't'

$$\frac{B_4}{B} = \frac{A_4}{A}$$

Integrating we get

$$B = k_1 A \qquad (19)$$

Where k_1 is a constant of integration.

The spatial volume for the model is given by.

$$\xi^3(t) = ABC \qquad (20)$$

$$\xi(t) = (ABC)^{\frac{1}{3}} \qquad (21)$$

Where ξ as the average scale factor, the hubble parameter and deceleration parameter are respectively definite.

$$H = \frac{\xi_4}{\xi} = \frac{1}{3}\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) \qquad (22)$$

$$H = \frac{1}{3}\left(H_1 + H_2 + H_3\right) \qquad (23)$$

Where $H_1 = \frac{A_4}{A}$, $H_2 = \frac{B_4}{B}$, $H_3 = \frac{C_4}{C}$ are the direction hubble parameter in x,y,z direction respectively.

$$g = -\frac{\xi\,\xi_{44}}{\xi_4^2} \qquad (24)$$

The physical quantities of the expansion scalar θ and shear tensor σ^2 are defined as

$$\theta = 3H = \frac{3\xi_4}{\xi}$$

$$\theta = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \qquad (25)$$

$$\sigma^2 = \sigma_{ij}\sigma^{ij} = \frac{1}{2}\left(\frac{A_4^2}{A^2} + \frac{B_4^2}{B^2} + \frac{C_4^2}{C^2}\right) - \frac{\theta^2}{6} \qquad (26)$$

The average an isotropy parameter A_m is given by

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 \qquad (27)$$

Where $\Delta H_i = H_i - H \qquad (i = 1,2,3)$

represent the directional Hubble parameter in x, y, z direction respectively and $A_m = 0$ corresponds to isotropic expansion.

from (19), we take $k_1 = 1$

$$B = (1)A$$

$$B = A$$

The field equ (14) to (17) can be written as

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC} = -8\pi G\left[\frac{(F_{23})^2e^{2nx}}{2B^2C^2}\right] \qquad (28)$$

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC} = 8\pi G\left[\frac{(F_{23})^2e^{2nx}}{2B^2C^2}\right] \qquad (29)$$

$$\frac{2B_{44}}{B} + \frac{B_4^2}{B^2} - \frac{n^2}{B^2} = 8\pi G\left[\eta + \frac{(F_{23})^2e^{2nx}}{2B^2C^2}\right] \qquad (30)$$

$$\frac{B_4^2}{B^2} + \frac{2B_4C_4}{BC} - \frac{n^2}{B^2} = 8\pi G\left[\rho - \frac{(F_{23})^2e^{2nx}}{2B^2C^2}\right] \qquad (31)$$

from equ (28) and (29) we get $F_{23} = 0$

The field equation (28) to (31) can be written as

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC} = 0 \qquad (32)$$

$$\frac{2B_{44}}{B} + \frac{B_4^2}{B^2} - \frac{n^2}{B^2} = 8\pi G\eta \qquad (33)$$

$$\frac{B_4^2}{B^2} + \frac{2B_4C_4}{BC} - \frac{n^2}{B^2} = 8\pi G\rho \qquad (34)$$

Solution of the field equations:

We assume that $\theta \propto \sigma$ which is physically condition. This condition leads to

$$C = B^m \qquad (35)$$

Where m is a constant

Massive strings:

In this case

$$\rho + \eta = 0 \qquad (36)$$

From equ (33), (34) and (36) we get

$$\frac{B_4 C_4}{BC} + \frac{B_4^2}{B^2} + \frac{B_{44}}{B} - \frac{n^2}{B^2} = 0 \qquad (37)$$

Using equation (35), the above equation can be written as

$$\frac{B_{44}}{B} + (m+1)\frac{B_4^2}{B^2} = \frac{n^2}{B^2} \qquad (38)$$

Now we put $B_4 = \Psi(B)$ in equ(38) we get

$$\frac{d}{dB}\Psi^2 + \frac{2(m+1)}{B}\Psi^2 = \frac{2n^2}{B} \qquad (39)$$

Integrating equ (38) we get

$$\Psi^2 = \left(\frac{dB}{dt}\right)^2 = \frac{n^2}{m+1} + k_2 B^{-2m-2} \qquad (40)$$

Where k_2 is integration constant

The Bianchi type III model in this case reduces to the form

$$ds^2 = -\left(\frac{dt}{dB}\right)^2 dB^2 + B^2[dx^2 + e^{-2nx}\, dy^2] + B^{2m} dz^2 \qquad (41)$$

Where $A = \tau, \quad x = X, \; y = Y, \; z = Z$

$$ds^2 = -\left[\frac{d\tau^2}{\frac{n^2}{m+1} k_2 B^{-2m-2}}\right] + \tau^2[dX^2 + e^{-2nx} dY^2] + \tau^{2m}\, dZ^2 \qquad (42)$$

Some physical and geometrical properties :

$$\rho = \frac{1}{8\pi}\left[\frac{mn^2}{(m+1)\tau^2} + \frac{(2m+1)k_2}{\tau^{2(m+2)}}\right] \qquad (43)$$

$$\eta = -\frac{1}{8\pi}\left[\frac{mn^2}{(m+1)\tau^2} + \frac{(2m+1)k_2}{\tau^{2(m+2)}}\right] \qquad (44)$$

Using (4) we get

$$\rho_p = \frac{1}{8\pi}\left[\frac{2mn^2}{(m+1)\tau^2} + \frac{2(2m+1)k_2}{\tau^{2(m+2)}}\right] \qquad (45)$$

$$\theta = (m+2)\left[\frac{n^2}{(m+1)\tau^2} + \frac{k_2}{\tau^{2(m+2)}}\right]^{\frac{1}{2}} \qquad (46)$$

$$\sigma^2 = \frac{(m-1)^2}{3}\left[\frac{n^2}{(m+1)\tau^2} + \frac{k_2}{\tau^{2(m+2)}}\right] \qquad (47)$$

$$\frac{\sigma^2}{\theta^2} = \frac{(m-1)^2}{3(m+2)^2}$$

$$\frac{\sigma}{\theta} = \frac{(m-1)}{\sqrt{3}(m+2)} \qquad (48)$$

The cloud string η and the particle density ρ_p vanish asymptotically in general if (m+2) $>$ 0 . The expansion in the model stops when n=-2. The model starts expanding with a big bang at $\tau = 0$ and the expansion in the model decreases as time increases if (m+2) $>$ 0. since $\lim_{\tau\to\infty}\frac{\sigma}{\theta}$= constant, the anisotropy is maintained for all time. It can be seen the model is irrotational. Therefore, the model describes a continuously expanding, shearing and non rotating universe with big bang.

Special Case :

when m=1 in equ (35) we get

$$B = C \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_(49)}$$

From equ (32) and (49) we get

$$\frac{2B_{44}}{B} + \frac{B_4^2}{B^2} = 0 \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_(50)}$$

Now we put $B_4 = \Psi(B)$ in equ (50)

We get

$$\frac{d}{dB}\Psi^2 + \frac{\Psi^2}{B} = 0 \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_(51)}$$

Integrating equ (51) we get

$$\Psi^2 = \left(\frac{dB}{dt}\right)^2 = \frac{k_3}{B} \qquad \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_(52)}$$

Where k_3 is integration constant

The Bianchi type III model in this case reduces to the form

$$ds^2 = -\left(\frac{dt}{dB}\right)^2 dB^2 + B^2[dx^2 + e^{-2nx}dy^2 + dz^2] \qquad \text{\_\_\_\_\_\_\_\_\_(53)}$$

Where A = , $x = X,\ y = Y,\ z = Z$

$$ds^2 = -\left(\frac{B}{k_3}\right) d\tau^2 + \tau^2\,[dX^2 + e^{-2nx}dY^2 + dZ^2] \qquad \text{\_\_\_\_\_\_\_\_\_\_\_(54)}$$

CONCLUSION

In this paper we have presented a new exact solution of Einstein’s field equations for Bianchi type III string cosmological model with electromagnetic in theory of relativity. In general the model is expanding, shearing and non rotating because constant. The model starts with a big bang at =0 and it goes on expanding until it comes out to rest at correspond to the proper time t=0 & t= respectively. since is constant in both cases, the models do not approach isotropy at any time geometrical and physical behaviour of the model is also discussed.

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