SOME BIANCHI TYPE-III BULK VISCOUS STRING COSMOLOGICAL MODELS WITH NON SHEAR IN MODIFIED GENERAL RELATIVITY

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ABSTRACT: - Here we investigated Bianchi type III string cosmological models with bulk viscosity. To get a deterministic model, it is assumed that $\eta \propto \theta$ where η is the coefficient of bulk viscosity, θ is the scalar of expansion. The physical and geometrical behaviours of the models are also discussed**.**

KEYWORDS: Bianchi type III cosmological model, string cosmological model, string, expansion scalar, bulk viscosity.

INTRODUCTION

Cosmic strings, in recent years. Have drawn considerable attention among researchers for various aspects such that the study of the early universe. The string theory is useful concept before the creation of the particle in the universe. the string are nothing but the important topological stable defects due to the phase transition that occurs as the temperature lower below some critical temperature at the very early stages of the universe.

The present day configuration of the universe is not contradicted by the large scale network of strings in the early universe. Moreover, the galaxy formation can be explained by the density fluctuations of the vacuum strings. The general relativistic treatment of strings was obtained by Letelier[1,2] and stachel[3]. Leterlier[1] has obtained the solution to Einstein's field equation for a cloud of strings with spherical plane and cylindrical symmetry. Then in 1983 he solved Einstein's field equation for a closed of massive strings and obtained cosmological model in Bianchi type I and Kantowski-sachs space times Baliet al [4-5] Tikerkar and Patel[6] and chakraborty and chakraborty7 presented exact solution for the Bianchi III Cosmological models for string cloud. The string models have an important rule in cosmology, as strings are believed to have an important role during the early stage of the universe[8] and can greate density fluctuations which lead to galaxy formation[9] Bianchi type III string cosmological model with viscosity where the constant coefficient of bulk viscosity is considered. Recently Bali and Prathan[10] have obtained formalism for studying the new interability of Bianchi type III massive strings cosmological models in genral relativity. Wang [11-12] has also discussed LRS Bianchi type I and Bianchi type III cosmological for a cloud string with bulk viscosity. Very recently, yadav, Rai and Pradhan[13] have found the integrability of cosmic string in Bianchi type III space time in presence of bulk viscous fluid by applying a new technique. Recently Tiwari and Sonia[14] investigated the non extence of shear in Bianchi type III string cosmological models with bulk viscosity and time dependent Λ.

To obtain a determine cosmological model we assume that the coefficient of the viscosity is proportional to the expansion scalar $\eta \propto \theta$. The Physical behaviours of models are also discussed.

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<u>Metric and field equation</u>:

We consider the Bianchi type- III metric of the from

$$ds^2 = -dt^2 + A^2 dx^2 + B^2(e^{2x}dy^2 + dz^2) \qquad \text{(1)}$$

Where A and B are function of t only.

The energy momentum tensor T_{ij} for the cloud of strings with bulk viscosity.

$$T_{ij} = \rho v_i v_j - \mu x_i x_j - \eta v^l_{;l}(v_i v_j + g_{ij}) \qquad \text{(2)}$$

$$\rho = \rho_p + \mu \qquad \text{(3)}$$

Where ρ the energy density for cloud of string with particle attached to them η is the coefficient of bulk viscosity, $\theta = v^l_{;l}$ is the scalar of expansion , ρ_p is the particle energy density, μ is the string tension density, v^i is the four velocity vector of particles and x^i is the unit space like vector representing the direction of string satisfying.

$$v_i v^i = -x_i x^i = -1, \qquad v^i x_i = 0 \tag{4}$$

In a co-moving system we get.

$$v^i = (0,0,0,1), \qquad x^i = \left(\frac{1}{A}, 0,0,0\right) \tag{5}$$

The expression for scalar of expansion θ and shear scalar σ are.

$$\theta = v^l_{;l} = \frac{A_4}{A} + \frac{2B_4}{B} = 3H \tag{6}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{3}\left(\frac{A_4^2}{A^2} + \frac{B_4^2}{B^2} - \frac{2A_4B_4}{AB}\right) \tag{7}$$

The spatial volume V and the average scale factor ξ(t) is given by.

$$V = \xi^3(t) = AB^2 \tag{8}$$

$$\xi(t) = (AB^2)^{\frac{1}{3}} \tag{9}$$

The Hubble parameter defined as

$$H = \frac{\xi_4}{\xi} = \frac{1}{3}\left(\frac{A_4}{A} + \frac{2B_4}{B}\right) \tag{10}$$

The Einstein is field equations with gravitational units $8\pi G = 1$ read as

$$R_{ij} - \frac{1}{2}Rg_{ij} = T_{ij} \tag{11}$$

for the metric (1), Einstein's field equation can be written.

$$\frac{2B_{44}}{B} + \frac{B_4^2}{B^2} = \eta\,\theta \tag{12}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4B_4}{AB} = \eta\,\theta \tag{13}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4B_4}{AB} - \frac{1}{A^2} = \mu + \eta\,\theta \tag{14}$$

$$\frac{2A_4B_4}{AB} + \frac{B_4^2}{B^2} - \frac{1}{A^2} = \rho \tag{15}$$

$$\frac{A_4}{A} - \frac{B_4}{B} = 0 \tag{16}$$

Here a suffix '4' indicate, an ordinary differ entiatio with respect to t.
From equ (16) we get.

$$\log A - \log B = \log n$$

$$\frac{A}{B} = n$$

$$A = nB \tag{17}$$

Where n is the constant of integration

From equ (17), we take n=1

A=(1)B

A=B ______________(18)

We assume that the coefficient of bulk viscosity is proportional to the expansion scalar θ i.e. $\eta \propto \theta$

$$\eta = e\theta \qquad (19)$$

Where 'e' is a positive constant. Therefore

$$\theta = \frac{3B_4}{B} = 3H \qquad (20)$$

$$\eta = e\theta = 3e\frac{B_4}{B} \qquad (21)$$

using equ (18), equ(12) ‾ (15) become.

$$\frac{2B_{44}}{B} + \frac{B_4^2}{B^2} = \eta\,\theta \qquad (22)$$

$$\frac{2B_{44}}{B} + \frac{B_4^2}{B^2} - \frac{1}{B^2} = \mu + \eta\,\theta \qquad (23)$$

$$\frac{3B_4^2}{B^2} - \frac{1}{B^2} = \rho \qquad (24)$$

Substituting equ (20) and (21) in to equ (22) we get.

$$\frac{2B_{44}}{B} + \frac{B_4^2}{B^2} = \left(3e\frac{B_4}{B}\right)\left(3\frac{B_4}{B}\right)$$

$$\frac{2B_{44}}{B} + \frac{B_4^2}{B^2} = 9e\frac{B_4^2}{B^2}$$

$$\frac{2B_{44}}{B} + (1-9e)\frac{B_4^2}{B^2} = 0$$

$$\frac{2B_{44}}{B} + \left(\frac{1-9e}{2}\right)\frac{B_4^2}{B^2} = 0$$

$$\frac{B_{44}}{B} + e_1\frac{B_4^2}{B^2} = 0 \qquad (25)$$

Where $e_1 = \frac{1-9e}{2}$

Integrating equ (25) we get

$$B^{e_1}\,dB = cdt$$

Again integrating we get

$$\frac{B^{(e_1+1)}}{(e_1+1)} = (ct+d)$$

$$B^{(e_1+1)} = (e_1+1)\,(ct+d)$$

$$B = (e_1+1)^{\frac{1}{(e_1+1)}} \quad (ct+d)^{\frac{1}{(e_1+1)}}$$

Where c and d are constant of integration Now

$$A = (e_1+1)^{\frac{1}{(e_1+1)}} \quad (ct+d)^{\frac{1}{(e_1+1)}}$$

Therefore the line element (1) can be written as

$$ds^2 = -dt^2 + \left[(e_1+1)^{\frac{1}{(e_1+1)}}(ct+d)^{\frac{1}{(e_1+1)}}\right]^2 (dx^2 + e^{2x}dy^2 + dz^2)$$

____________(26)

$$= \frac{B_4}{B} = \frac{c}{(e_1+1)(ct+d)}$$

Some Physical and Geometrical Properties

The energy density ρ, the string tension density μ, coefficient of bulk viscosity η, the scalar of expansion θ, the shear scalar σ, ρ_p the particle energy density are respectively given by

$$\theta = \frac{3c}{(e_1+1)(ct+d)}$$

________________(27)

$$\rho = \frac{3c^2}{(e_1+1)^2(ct+d)^2} - \frac{1}{(e_1+1)^{\frac{2}{e_1+1}}(ct+d)^{\frac{2}{e_1+1}}}$$

$$\mu = -(e_1+1)^{\frac{-2}{e_1+1}}(ct+d)^{\frac{-2}{e_1+1}}$$

$$\sigma^2 = 0$$

$$\sigma = 0$$

DISCUSSION

In this paper we have studied Bianchi type III string Cosmotogical model with bulk viscosity. We adopt the condition $\eta \propto \theta$. Then the cosmological model for string cosmology with bulk viscosity and cosmological term is obtained. The energy density $\rho \to \infty$ when t$\to 0$ and $\rho \to 0$ when $t \to \infty$ therefore the model describes a shearing, non rotating continously expanding universe with a big bang start. As the time t increase the rate of expansion θ decreases. In the absence of bulk viscosity i.e.

$\eta = 0$ we get

$\frac{\sigma}{\theta} = 0$ therefore model approach isotropy for large value of t.

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